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Dynamic analysis of spherical shell partially filled with fluid

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ABSTRACT

In present study, a hybrid finite element method is applied to investigate the free vibration of spherical shell filled with fluid. The structural model is based on a combination of thin shell theory and the classical finite element method. It is assumed that the fluid is incompressible and has no free-surface effect. Fluid is considered as a velocity potential variable at each node of the shell element where its motion is expressed in terms of nodal elastic displacement at the fluid–structure interface. Numerical simulation is done and vibration frequencies for different filling ratios are obtained and compared with existing experimental and theoretical results. The dynamic behavior for different shell geometries, filling ratios and boundary conditions with different radius to thickness ratios is summarized. This proposed hybrid finite element method can be used efficiently for analyzing the dynamic behavior of aerospace structures at less computational cost than other commercial FEM software.

1. Introduction

Shells of revolution, particularly spherical shells are one of the primary structural elements in high speed aircraft. Their applications include the propellant tank or gas-deployed skirt of space crafts. Space shuttles need a large thrust within a short time interval; thus a large propellant tank is required. Dynamic behavior in the lightweight, thin-walled tank is an important aspect in its design. These liquid propelled space launch vehicles experience a significant longitudinal disturbance during thrust build up and also due to the effect of launch mechanism. Dynamic analysis of such a problem in the presence of fluid–structure interaction is one of the challenging subjects in aerospace engineering. Great care must be taken during the design of spacecraft vehicles to prevent dynamic instability.

Free vibration of spherical shell containing a fluid has been investigated by numerous researchers experimentally and analytically. Rayleigh [1] solved the problem of axisymmetric vibrations of a fluid in a rigid spherical shell. The solution for vibrations of the fluid-filled spherical membrane appears in the work of Morse and Feshbach [2].

The fluid movement on the surface of fluid (sloshing) in non-deformable spherical shell has been investigated by few researchers as Budiansky [3], Stofan and Armsted [4], Chu [5], Karamanos et al. [6]. The oscillations of the fluid masses result from the lateral displacement or angular rotation of the spherical shell. Others researchers have studied particular cases like the case of a sphere filled with fluid. Rand and Dimaggio [7] considered the free vibrations for axisymmetric, extensional, non-torsional of fluid-filled elastic spherical shells. Motivated by the fact that human head can be represented as a spherical shell filled by fluid, Engin and Liu [8] considered the free vibration of a thin homogenous spherical shell containing an inviscid irrotational fluid. Advani and Lee [9] investigated the vibration of the fluid-filled shell using higher-order shell theory including transverse shear and rotational inertia. Guarino and Elger [11] have looked at the frequency spectra of a fluid-filled sphere, both with and without a central solid sphere, in order to explore the use of auscultatory percussion as a clinical diagnostic tool. Free vibration of a thin spherical shell filled with a compressible fluid is investigated by Bai and Wu [12]. The general non-axisymmetric free vibration of a spherically isotropic elastic spherical shell filled with a compressible fluid medium has been investigated by Chen and Ding [13]. Young [14] studied the free vibration of spheres composed of inviscid compressible liquid cores surrounded by spherical layers of linear elastic, homogeneous and isotropic materials.

The case of hemispherical shells filled with fluid was studied experimentally by Samoilov and Pavlov [15]. Hwang [16] investigated the case of the longitudinal sloshing of liquid in a flexible hemispherical tank supported along the edge. Chung and Rush [17] presented a rigorous and consistent formulation of dynamically coupled problems dealing with motion of a surface-fluid-shell system. A numerical example of a hemispherical bulkhead filled with fluid is modeled.

Komatsu [18,19] used a hybrid method with a fluid mass coefficient added to his system of equations. He also validated his model with experiments on hemispherical shells partially filled

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with fluid under two boundary conditions: a clamped boundary condition and a free boundary condition. Recently, Ventsel et al. [20] used a combined formulation of the boundary elements method and finite elements method to study the free vibration of an isotropic simply supported hemispherical shell with different circumferential mode numbers.

For a spherical shell that is partially liquid-filled, if one wishes to consider the hydroelastic vibration developed as consequence of interaction between hydrodynamic pressure of liquid and elastic deformation of the shell, this is a complex problem. Numerical method such as the finite element method (FEM) are therefore used since they are powerful tools that can adequately describe the dynamic behavior of such system which contains complex structures, boundary conditions, materials and loadings. Some powerful commercial FEM software exists, such as ANSYS, ABAQUS and NASTRAN. When using these tools to model such a complex FSI problem, a large numbers of elements are required in order to get good convergence. The hybrid approach presented in this study provides very fast and precise convergence with less numerical cost compared to these commercial software packages.

In this work a combined formulation of shell theory and the hybrid finite element method (FEM) is applied to model the shell structure. Nodal displacements are found from exact solution of shell theory. This hybrid FEM has been applied to produce efficient and robust models during analysis of both cylindrical and conical shells. A spherical shell which has been filled partially with incompressible and inviscid is modeled in this study. The fluid is characterized as a velocity potential variable at each node of the shell finite element mesh; then fluid and structures are coupled through the linearized Bernoulli's equation and impermeable boundary condition at the fluid–structure interface. Dynamic analysis of the structure under various geometries, boundary conditions and filling ratios is analyzed.

2. Formulation

2.1. Structural modeling

In this study the structure is modeled using hybrid finite element method which is a combination of spherical shell theory and classical finite element method. In this hybrid finite element method, the displacement functions are found from exact solution of spherical shell theory rather approximated by polynomial functions as is done in classical finite element method. In the spherical coordinate system (R, θ, ϕ) shown in Fig. 1, five out of the six equations of equilibrium derived in reference for spherical shells are written as follows:

$$\begin{aligned}
&\frac{\partial N_\phi}{\partial \phi} + \frac{1}{\sin\phi}\frac{\partial N_{\phi\theta}}{\partial \theta} + (N_\phi - N_\theta)\cot\phi + Q_\phi = 0\\
&\frac{\partial N_{\phi\theta}}{\partial \phi} + \frac{1}{\sin\phi}\frac{\partial N_\phi}{\partial \theta} + 2N_{\phi\theta}\cot\phi + Q_\theta = 0\\
&\frac{\partial Q_\phi}{\partial \phi} + \frac{1}{\sin\phi}\frac{\partial Q_\phi}{\partial \theta} + Q_\phi\cot\phi - (N_\phi + N_\theta) = 0\\
&\frac{\partial M_\phi}{\partial \phi} + \frac{1}{\sin\phi}\frac{\partial M_{\phi\theta}}{\partial \theta} + (M_\phi - M_\theta)\cot\phi + RQ_\phi = 0\\
&\frac{\partial M_{\phi\theta}}{\partial \phi} + \frac{1}{\sin\phi}\frac{\partial M_\phi}{\partial \theta} + 2M_{\phi\theta}\cot\phi + RQ_\theta = 0
\end{aligned} \tag{1}$$

where N_ϕ, N_θ, $N_{\phi\theta}$ are membrane stress resultants (forces per unit of length of the middle surface); M_ϕ, M_θ, $M_{\phi\theta}$ the bending stress resultants(moments per unit of length of the middle surface) and Q_ϕ, Q_θ the shear forces(forces per unit of length of the middle surface) (Fig. 2). The sixth equation, which is an identity equation for spherical shells, is not presented here.

Strains and displacements for three displacements in meridional U_ϕ, radial W and circumferential U_θ are related as follows:

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_\phi \\ \varepsilon_\theta \\ 2\varepsilon_{\phi\theta} \\ \kappa_\phi \\ \kappa_\theta \\ 2\kappa_{\phi\theta} \end{Bmatrix} = \begin{Bmatrix} \frac{1}{R}\left(\frac{\partial U_\phi}{\partial \phi} + W\right) \\ \frac{1}{R}\left(\frac{1}{\sin\phi}\frac{\partial U_\theta}{\partial \theta} + U_\phi\cot\phi + W\right) \\ \frac{1}{R}\left(\frac{\partial U_\theta}{\partial \phi} + \frac{1}{\sin\phi}\frac{\partial U_\phi}{\partial \theta} - U_\theta\cot\phi\right) \\ \frac{1}{R^2}\left(\frac{\partial U_\phi}{\partial \phi} - \frac{\partial^2 W}{\partial \phi^2}\right) \\ \frac{1}{R^2}\left(\frac{1}{\sin\phi}\frac{\partial U_\theta}{\partial \theta} + U_\phi\cot\phi - \frac{1}{\sin^2\phi}\frac{\partial^2 W}{\partial \theta^2} - \cot\phi\frac{\partial W}{\partial \phi}\right) \\ \frac{1}{R^2}\left(\frac{\partial U_\theta}{\partial \phi} + \frac{1}{\sin\phi}\frac{\partial U_\phi}{\partial \theta} - U_\theta\cot\phi + 2\frac{1}{\sin\phi}\cot\phi\frac{\partial W}{\partial \theta} - 2\frac{1}{\sin\phi}\frac{\partial^2 W}{\partial \phi\partial \theta}\right) \end{Bmatrix} \tag{2}$$

The displacements U, W and V in the global Cartesian coordinate system are related to displacements $U_{\phi i}$, W_i and $U_{\theta i}$ indicated in Fig. 3. by:

$$\begin{Bmatrix} U \\ W \\ V \end{Bmatrix} = \begin{bmatrix} \sin\phi_i & -\cos\phi_i & 0 \\ \cos\phi_i & \sin\phi_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} U_{\phi i} \\ W_i \\ U_{\theta i} \end{Bmatrix} \tag{3}$$

The membrane stress resultants and bending stress resultants vector $\{\sigma\} = \{N_\phi \quad N_\theta \quad N_{\phi\theta} \quad M_\phi \quad M_\theta \quad M_{\phi\theta}\}^T$ is expressed as function of strain $\{\varepsilon\}$ by

$$\{\sigma\} = [P]\{\varepsilon\} \tag{4}$$

where $[P]$ is the elasticity matrix for an anisotropic shell given by:

$$[P] = \begin{bmatrix} P_{11} & P_{12} & 0 & P_{14} & P_{15} & 0 \\ P_{21} & P_{22} & 0 & P_{24} & P_{25} & 0 \\ 0 & 0 & P_{33} & 0 & 0 & P_{36} \\ P_{41} & P_{42} & 0 & P_{44} & P_{45} & 0 \\ P_{51} & P_{52} & 0 & P_{54} & P_{55} & 0 \\ 0 & 0 & P_{63} & 0 & 0 & P_{66} \end{bmatrix} \tag{5}$$

Upon substitution of Eqs. (2), (4) and (5) into Eq. (1), a system of equilibrium equations can be obtained as a function of displacements:

$$\begin{aligned}
&L_1(U_\phi, W, U_\theta, P_{ij}) = 0\\
&L_2(U_\phi, W, U_\theta, P_{ij}) = 0\\
&L_3(U_\phi, W, U_\theta, P_{ij}) = 0
\end{aligned} \tag{6}$$

These three linear partial differentials operators L_1, L_2 and L_3 are given in Appendix A, and P_{ij} are elements of the elasticity matrix which, for an isotropic thin shell with thickness h is given by:

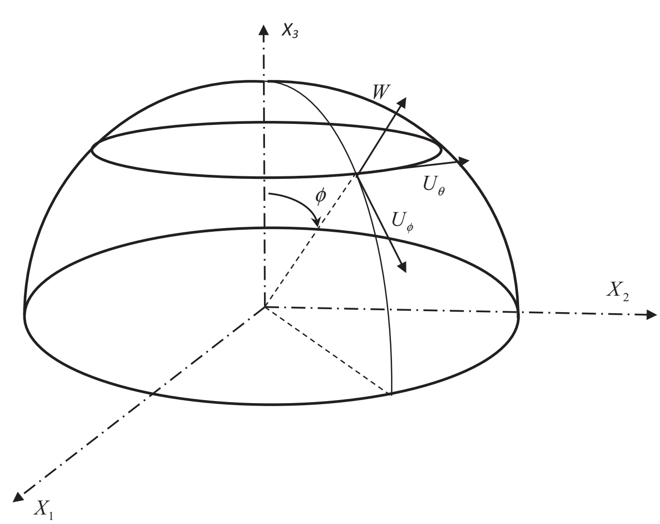

Fig. 1. Geometry of spherical shell.