Computers & Fluids 108 (2015) 179–185

Contents lists available at ScienceDirect

Computers & Fluids

journal homepage: www.elsevier.com/locate/compfluid

MHD stagnation-point flow of Jeffrey fluid over a convectively heated stretching sheet

T. Hayat [a,b], Sadia Asad [a,*], M. Mustafa [c], A. Alsaedi [b]

[a] Department of Mathematics, Quaid-i-Azam University 45320, Islamabad 44000, Pakistan
[b] Nonlinear Analysis and Applied Mathematics (NAAM), Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[c] School of Natural Sciences (SNS), National University of Sciences and Technology (NUST), Sector H-12, Islamabad 44000, Pakistan

ARTICLE INFO

Article history:
Received 29 August 2013
Received in revised form 19 September 2014
Accepted 11 November 2014
Available online 26 November 2014

Keywords:
Viscoelastic flow
Boundary layer flow
Exponentially stretching sheet
Convective boundary condition
Magnetohydrodynamics
Thermal radiation

ABSTRACT

Two-dimensional stagnation-point flow of Jeffrey fluid over an exponentially stretching sheet is studied. Convective boundary condition is used for the analysis of thermal boundary layer. In addition the combined effects of thermal radiation and magnetic field are taken into consideration. The developed nonlinear problems have been solved for the series solution. The convergence of the series solutions is carefully analyzed. The behaviors of various physical parameters such as viscoelastic parameter (β), magnetic field parameter (M), radiation parameter (R), Biot number (Bi) and velocity ratio parameter (α) are examined through graphical and numerical results of velocity and temperature distributions.

1. Introduction

The study of flow and heat transfer over continuously moving extensible or inextensible surfaces have received great attention in the past due to their various industrial and engineering applications. For example the production of sheeting material is involved in manufacturing processes and includes both metal and polymer sheets. Here the performance of heat transfer at the sheet has a pivotal role on the quality of final product. Industrial relevant applications include fibers spinning, hot rolling, manufacturing of plastic and rubber sheets, continuous casting and glass blowing. Flow over a flat plate with uniform free stream has been discussed by Blasius [1]. Sakiadis [2] initially studied the boundary layer flow over a continuously moving flat plate in a quiescent ambient fluid. Crane [3] discussed the flow over an extensible sheet which is stretched in its own plane with the velocity proportional to the distance from the origin. Perturbation solutions for flow of viscoelastic fluid bounded by a stretching sheet have been obtained by Rajagopal et al. [4]. Sankara and Watson [5] and Andersson et al. [6] have extended the Crane's problem for micropolar and power-law fluids respectively. Mahapatra and Gupta [7] obtained analytic solution for flow of second grade fluid over a stretching sheet by regular perturbation method. Liu [8] derived the exact solutions for flow with heat and mass transfer of viscous fluid with internal heat generation and chemical reaction. Cortell [9] numerically examined the characteristics of mass transfer in the two-dimensional flow of viscoelastic fluid over a stretching sheet. Influences of viscous dissipation and thermal radiation on the flow past a stretching sheet has been investigated by Cortell [10]. Hayat et al. [11] analyzed the characteristics of heat and mass transfer in the flow of second grade fluid over a stretching sheet. Melting heat transfer in the stagnation-point flow over a stretching/shrinking sheet has been addressed by Bachok et al. [12]. MHD stagnation-point flow of power-law fluid over a stretching surface has been discussed by Mahapatra et al. [13]. Heat transfer over an impermeable stretching sheet with non-uniform heat source/sink has been investigated by Nandeppanavar et al. [14]. Recently various contributions dealing with the MHD boundary layer flow over stretching surfaces have been reported (see Ellahi and Riaz [15], Hameed and Ellahi [16,17], Ellahi [18], Zeeshan and Ellahi [19]). The above mentioned studies were only confined to the flows over a linearly stretching surface. However in industrial applications the sheet may be stretched in a variety of ways. In this regard the flow analysis dealing with the exponentially stretching sheet is scarcely addressed. Simultaneous effects of heat and mass transfer in the boundary layer flow over an exponentially stretching sheet has been reported by Magyari and Keller [20]. Suction and heat transfer

* Corresponding author. Tel.: +92 51 90642172.
E-mail address: asadsadia@ymail.com (A. Alsaedi).

http://dx.doi.org/10.1016/j.compfluid.2014.11.016
0045-7930/

characteristics in the exponentially stretched flow have been investigated by Elbashbeshy [21]. Viscoelastic effects in the flow over an exponentially stretching sheet have been examined by Khan and Sanjayanand [22]. Sajid and Hayat [23] provided homotopy solutions for thermal radiation effect in the flow by an exponentially stretching sheet. Nadeem et al. [24] explored the flow and heat transfer of viscoelastic (second grade) fluid over an exponentially sheet in the presence of thermal radiation.

Convective heat transfer with thermal radiation is involved in various engineering processes including thermal energy storage, gas turbines, nuclear turbines, die forging and chemical reactions. Aziz [25] numerically investigated the viscous flow over a flat plate with convective boundary conditions. Magyari [26] computed the exact solution to this problem in a compact integral form. Radiation effects in the Blasius and Sakiadis flows with convective boundary conditions have been described by Cortell [27]. Ishak [28] provided the numerical solution for flow and heat transfer over a permeable stretching sheet with convective boundary conditions. Yao et al. [29] obtained a closed form exact solution for viscous flow over a permeable stretching/shrinking convectively heated wall. Series solutions for flows of Jeffrey and second grade fluids with convective boundary conditions have been computed by Hayat et al. [30,31]. In another investigation Alsaedi et al. [32] computed exact solutions for steady flow of Jeffrey fluid over a linearly stretching surface with convective boundary conditions. Makinde and Aziz [33] considered the analysis of convective heat transfer in the steady flow of nanofluid. Mustafa et al. [34] recently explored the axisymmetric flow of nanofluid over a convectively heated radially stretching sheet.

The present work is concerned with the development of momentum and thermal boundary layers for the flow of viscoelastic Jeffrey fluid over an exponentially stretching sheet. Heat transfer analysis is performed in the presence of thermal radiation and convective boundary conditions. The series expressions of velocity and temperature functions are constructed by homotopy analysis method (HAM) [35–40]. Plots for the influence of embedded flow quantities on the velocity and temperature are displayed and discussed in detail.

2. Problem formulation

We consider the steady two-dimensional MHD stagnation point flow of an incompressible Jeffrey fluid over an exponentially stretching sheet. The magnetic field strength B_0 is applied normal to the plate. Induced magnetic field is assumed to be negligible in comparison with applied magnetic field for small magnetic Reynolds number. In addition, heat transfer analysis is considered with radiation effects. Further we consider x-axis parallel to the sheet and y-axis normal to it (see Fig. 1).

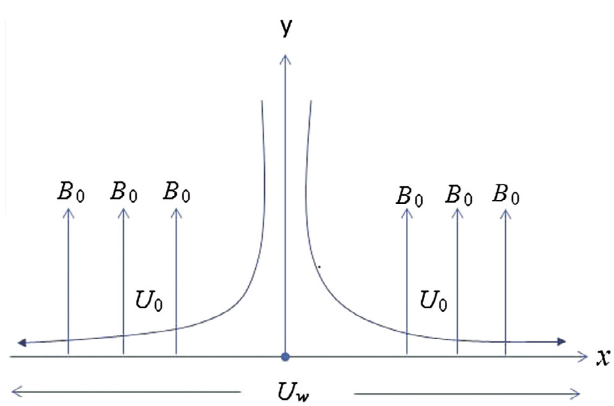

Fig. 1. Physical model and coordinate system.

The velocity and temperature fields subject to boundary layer approximations are governed by the following equations

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_0\frac{dU_0}{dx}+\frac{\upsilon}{1+\lambda}\left[\frac{\partial^2 u}{\partial y^2}+\lambda_1\begin{pmatrix}\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+u\frac{\partial^3 u}{\partial x\partial y^2}\\ -\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+v\frac{\partial^3 u}{\partial y^3}\end{pmatrix}\right]-\frac{\sigma B_0^2}{\rho}(u-U_0), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\sigma\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}, \tag{3}$$

$$u=U_w(x)=ce^{\frac{x}{L}},\ \ v=0,\ \ -k\frac{\partial T}{\partial y}=h(T_f-T)\ \text{at}\ y=0 \tag{4}$$

$$u=U_0(x)=ae^{\frac{x}{L}},\ \ T=T_\infty\ \text{as}\ y\to\infty, \tag{5}$$

where u and v represent the velocity components along x and y axes respectively, U_0 is the strain velocity, λ is the ratio of relaxation to retardation times, λ_1 is the retardation time, U_w is the stretching/shrinking velocity, T is the fluid temperature, σ is the thermal diffusivity of the fluid, $v=(\mu/\rho)$ is the kinematic viscosity, ρ is the density of fluid, k is the thermal conductivity of fluid, h is the convective heat transfer coefficient and T_f is the convective fluid temperature below the moving sheet.

We define the similarity transformations as

$$\psi=\sqrt{2avL}f(\eta)e^{x/2L},\ \ \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty},\ \ \eta=y\sqrt{\frac{a}{2vL}}e^{x/2L}, \tag{6}$$

where a and L are constants and prime denotes the differentiation with respect to η,f is the dimensionless stream function, θ is the dimensionless temperature and ψ is the stream function given by

$$u=\frac{\partial\psi}{\partial y},\ \ v=-\frac{\partial\psi}{\partial x}.$$

Now Eq. (1) is identically satisfied and (2)–(5) yield

$$f'''+(1+\lambda)(ff''-2f'^2+2)+\beta\left(\frac{3}{2}f''^2+f'f'''-\frac{1}{2}ff^{iv}\right)-2(1+\lambda)M^2(f'-1)=0, \tag{7}$$

$$\left(1+\frac{4}{3}R\right)\theta''+\Pr(f\theta')=0, \tag{8}$$

$$f=0,\ f'=c/a=\alpha,\ \ \theta'=-Bi[1-\theta(0)]\ \text{at}\ \eta=0, \tag{9}$$

$$f'=1,\ \ \theta=0\ \text{at}\ \eta=\infty, \tag{10}$$

where $\beta=\frac{\lambda_1 ae^{x/L}}{L}$ is a local dimensionless parameter, $\alpha=\frac{c}{a}$ is a ratio parameter with $\alpha>1$ for assisting flow and $\alpha<1$ for opposing flow (i.e. when free stream velocity exceeds the stretching velocity), $Pr=\frac{v}{\sigma}$ is the Prandtl number, $R=\frac{4\sigma^* T_\infty^3}{kk^*}$ is a radiation parameter and $Bi=\frac{h}{k}\sqrt{\frac{v}{a}}$ is the Biot number. The skin friction coefficient C_f and local Nusselt number Nu_x are

$$C_f=\frac{\tau_w}{\frac{1}{2}\rho U_w^2},\ \ Nu_x=\frac{xq_w}{k(T_f-T_\infty)}, \tag{11}$$

where the skin friction τ_w and the heat transfer from the plate q_w are