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Optimal feedback control for semilinear fractional evolution equations in Banach spaces☆

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ABSTRACT

In this paper, we study optimal feedback controls of a system governed by semilinear fractional evolution equations via a compact semigroup in Banach spaces. By using the Cesari property, the Fillippove theorem and extending the earlier work on fractional evolution equations, we prove the existence of feasible pairs. An existence result of optimal control pairs for the Lagrange problem is presented.

1. Introduction

Fractional differential equations have been proved to be one of the most effective tools in the modeling of many phenomena in various fields of physics, mechanics, chemistry, engineering, etc. They have a great number of applications in nonlinear oscillations of earthquakes, many physical phenomena such as seepage flow in porous media and in the fluid dynamic traffic model. For more details, see the monographs of Kilbas et al. [1], Lakshmikantham et al. [2], Miller and Ross [3], Podlubny [4], Tarasov [5], and the survey of Agarwal et al. [6,7]. Recently, fractional differential equations and their optimal control have been studied by many researchers including us (see, for instance, [8–24] and the references therein).

On the other hand, there could be no manufacturing, no vehicles, no computers and no regulated environment without control systems. Control systems are most often based on the principle of feedback, whereby the signal to be controlled is compared to a desired reference signal and the discrepancy used to compute corrective control action [25,26]. Optimal feedback control of semilinear evolution equations in Banach spaces has been studied [27,28]; however, optimal feedback control of fractional evolution equations in Banach spaces has not been studied extensively.

Motivated by our previous work [19,20,23,24,28], we consider optimal feedback control of a system governed by the following semilinear fractional evolution equations:

$$\begin{cases} {}^{C}D_t^q x(t) = Ax(t) + f(t, x(t), u(t)), \\ \quad t \in J = [0, T],\ q \in (0, 1), \\ x(0) = x_0, \end{cases} \tag{1}$$

where ${}^{C}D_t^q$ is the Caputo fractional derivative of order $q \in (0, 1)$, and $A : D(A) \to X$ is the infinitesimal generator of a compact C_0-semigroup $\{T(t), t \geq 0\}$ in a reflexive Banach space X. The control u takes its value from $U[0, T]$, which is a control set which will be introduced in Section 3, and $f : J \times X \times U \to X$ will be specified in the latter.

To achieve our purpose, we firstly give the existence of mild solutions for the system (1). Secondly, we prove the existence result of feasible pairs involving the compactness of operators with the help of the Cesari property and the Fillippove theorem. Then, we present the existence of optimal feedback controls for the Lagrange problem (P). We remark that the system (1) is more complex than the classical first order semilinear evolution equation because a fractional derivative has appeared. After overcoming some difficulty from the Caputo fractional derivative, we extend the classical results on optimal feedback controls to the case of semilinear fractional evolution equations.

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The rest of the paper is organized as follows. In Section 2, some notations and preparation results are introduced. In Section 3, the existence of mild solutions and feasible pairs for the system (1) are presented. Finally, the existence of optimal feedback controls for the Lagrange problem (P) is proved.

2. Preliminaries

Throughout this paper, we denote by X a reflexive Banach space and by U a Polish space which is a separable completely meritable topological space. Let $C(J, X)$ be the Banach space of continuous functions from J to X with the usual supremum norm. For $1 < p < +\infty$, the Banach space $L^p(J, X)$ consists of the usual strongly measurable X-valued functions having p-th power summable norms. Suppose that $A : D(A) \to X$ is the infinitesimal generator of a compact C_0-semigroup $\{T(t), t \geq 0\}$. This means that there exists $M > 0$ such that $\sup_{t\in J} \|T(t)\| \leq M$. By

$$O_r(x) = \{y \in X \mid \|y - x\| \leq r\}$$

we denote the ball centered at x with the radius $r > 0$.

Definition 2.1 (*[29]*). Let E and F be two metric spaces. A multifunction $\mathcal{F} : E \to 2^F$ is said to be pseudo-continuous at $t \in E$ if

$$\bigcap_{\epsilon>0} \overline{\mathcal{F}(O_\epsilon(t))} = \mathcal{F}(t).$$

We say that $\mathcal{F}$ is pseudo-continuous on E if it is pseudo-continuous at each point $t \in E$.

We recall some basic definitions and properties of the fractional calculus theory which are used further in this paper. For more details, see [1].

Definition 2.2. The fractional integral of order γ with the lower limit zero for a function f is defined as

$$I^\gamma f(t) = \frac{1}{\Gamma(\gamma)} \int_0^t \frac{f(s)}{(t-s)^{1-\gamma}} ds, \quad t > 0,\ \gamma > 0,$$

provided the right side is point-wise defined on $[0, \infty)$, where $\Gamma(\cdot)$ is the gamma function.

Definition 2.3. The Riemann–Liouville derivative of order γ with the lower limit zero for a function $f : [0, \infty) \to R$ can be written as

$$^L D^\gamma f(t) = \frac{1}{\Gamma(n-\gamma)} \frac{d^n}{dt^n} \int_0^t \frac{f(s)}{(t-s)^{\gamma+1-n}} ds,$$
$$t > 0,\ n-1 < \gamma < n.$$

Definition 2.4. The Caputo derivative of order γ for a function $f : [0, \infty) \to R$ can be written as

$$^C D^\gamma f(t) = {}^L D^\gamma \left(f(t) - \sum_{k=0}^{n-1} \frac{t^k}{k!} f^{(k)}(0) \right), \quad t > 0,\ n-1 < \gamma < n.$$

Remark 2.5. (i) If $f(t) \in C^n[0, \infty)$, then

$$^C D^\gamma f(t) = \frac{1}{\Gamma(n-\gamma)} \int_0^t \frac{f^{(n)}(s)}{(t-s)^{\gamma+1-n}} ds = I^{n-\gamma} f^{(n)}(t), \quad t > 0,\ n-1 < \gamma < n.$$

(ii) The Caputo derivative of a constant is equal to zero.

(iii) If f is an abstract function with values in X, then integrals which appear in Definitions 2.2 and 2.3 are taken in Bochner's sense.

3. Existence of feasible pairs for fractional evolution equations

We make the following assumptions.

[S] : X is a reflexive Banach space and U is a Polish space.

[A] : A is the infinitesimal generator of a compact C_0-semigroup $\{T(t), t \geq 0\}$ on X.

[F1] : $f : J \times X \times U \to X$ is Borel measurable in (t, x, u) and is continuous in (x, u).

[F2] : f satisfies local Lipschitz continuity with respect to x, i.e., for any constant $\rho > 0$, there is a constant $L(\rho) > 0$ such that

$$\|f(t, x_1, u) - f(t, x_2, u)\| \leq L(\rho) \|x_1 - x_2\|,$$

for every $x_1, x_2 \in X$, $t \in J$, and uniformly $u \in U$ provided with $\|x_1\|, \|x_2\| \leq \rho$.

[F3] : For arbitrary $t \in J$, $x \in X$ and $u \in U$, there exists a positive constant $M > 0$ such that

$$\|f(t, x, u)\| \leq M(1 + \|x\|).$$

[F4] : For almost all $t \in J$, the set $f(t, x, \mathcal{F}(t, x))$ satisfies the following:

$$\bigcap_{\delta>0} \overline{co} f(t, O_\delta(x), \mathcal{F}(O_\delta(t, x))) = f(t, x, \mathcal{F}(t, x)).$$

[U] : $\mathcal{F} : J \times X \to 2^U$ is pseudo-continuous.

Let

$$U[0, T] = \{u : J \to U \mid u(\cdot) \text{ is measurable}\}.$$

Then, any element in the set $U[0, T]$ is called a control on J.

Based on our previous work, we introduce the following definition of mild solutions for the system (1).

Definition 3.1 (*Lemma 3.1 and Definition 3.1 [24]*). A mild solution $x \in C(J, X)$ of the system (1) is defined as a solution of the following integral equation:

$$x(t) = \mathscr{T}(t)x_0 + \int_0^t (t-\theta)^{q-1} \mathscr{S}(t-\theta) f\big(\theta, x(\theta), u(\theta)\big) d\theta, \quad t \in J, \quad (2)$$

where

$$\mathscr{T}(t) = \int_0^\infty \xi_q(\theta) T(t^q\theta) d\theta,$$

$$\mathscr{S}(t) = q \int_0^\infty \theta \xi_q(\theta) T(t^q\theta) d\theta,$$

$$\xi_q(\theta) = \frac{1}{q} \theta^{-1-\frac{1}{q}} \varpi_q\left(\theta^{-\frac{1}{q}}\right) \geq 0,$$

$$\varpi_q(\theta) = \frac{1}{\pi} \sum_{n=1}^\infty (-1)^{n-1} \theta^{-qn-1} \frac{\Gamma(nq+1)}{n!} \sin(n\pi q),$$
$$\theta \in (0, \infty),$$

ξ_q is a probability density function defined on $(0, \infty)$, that is

$$\xi_q(\theta) \geq 0, \quad \theta \in (0, \infty) \quad \text{and} \quad \int_0^\infty \xi_q(\theta) d\theta = 1.$$

Any solution $x(\cdot) \in C(J, X)$ of the system (1) is referred to as a state trajectory of the fractional evolution equation corresponding to the initial state x_0 and the control $u(\cdot)$.

The following lemma will be widely used in the following. For the reader's convenience, we state it here.

Lemma 3.2 (*Lemmas 3.2–3.4, [24]*). *The operators $\mathscr{T}$ and $\mathscr{S}$ have the following properties.*