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Probability-guaranteed H_∞ finite-horizon filtering for a class of nonlinear time-varying systems with sensor saturations ☆

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A B S T R A C T

In this paper, the probability-guaranteed H_∞ finite-horizon filtering problem is investigated for a class of nonlinear time-varying systems with uncertain parameters and sensor saturations. The system matrices are functions of mutually independent stochastic variables that obey uniform distributions over known finite ranges. Attention is focused on the construction of a time-varying filter such that the prescribed H_∞ performance requirement can be guaranteed with probability constraint. By using the difference linear matrix inequalities (DLMIs) approach, sufficient conditions are established to guarantee the desired performance of the designed finite-horizon filter. The time-varying filter gains can be obtained in terms of the feasible solutions of a set of DLMIs that can be recursively solved by using the semi-definite programming method. A computational algorithm is specifically developed for the addressed probability-guaranteed H_∞ finite-horizon filtering problem. Finally, a simulation example is given to illustrate the effectiveness of the proposed filtering scheme.

1. Introduction

Due to its clear engineering significance, the filtering problem has attracted a great deal of research attention in the past few decades. The filtering theory that has been successfully applied in many branches of engineering systems such as target tracking, mobile robot localization, and computer vision. A rich body of literature has appeared on the general filtering problem with a variety of performance requirements, see e.g., [1–10]. It is well known that sensors may not always produce signals of unlimited amplitude due mainly to the physical constraints or technological restrictions. The sensor saturation, if not properly handled, will inevitably affect the implementation precision of the designed filtering/control algorithms and may even cause undesirable degradation of the filter/controller performance. Consequently, the sensor saturation problem has been gaining an increasing research interest, see e.g., [11–14]. It is worth mentioning that, because of the mathematical complexity, most existing results concerning the sensor saturations have been concerned with *time-invariant systems over the infinite-horizon*. Unfortunately, in reality, almost all real-time systems should be time-varying especially those after digital discretization. Recently, motivated by the practical importance of the sensor saturation issues, the set-membership filtering problem has been investigated in [13] for a class of time-varying systems with saturated sensors.

In traditional control theory, the performance objectives of a controlled system are usually required to be met accurately. However, for many stochastic control problems, due to a variety of unpredictable disturbances, it is neither possible nor necessary to enforce the system performance with probability 1. Instead, it is quite common for practical control systems to attain their individual performance objective with certain satisfactory probability. These kinds of engineering problems have given rise to great challenges for the realization of multiple control objectives with respect to individual probability constraints. In particular, as a newly emerged research topic, the probability-guaranteed H_∞ controller design problem has been raised in [15] and then thoroughly investigated in [16–19] in an elegant way. Despite the advances made on the research topic of probability-guaranteed design, there is still much room for further investigation on more comprehensive systems in order to cover more engineering practice. For example, in reality, most engineering systems are nonlinear and time-varying with saturated sensors, where

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the performances are usually evaluated over a finite-horizon for time-varying systems. It is, therefore, the purpose of this paper to address the probability-guaranteed H_∞ finite-horizon filtering problem for nonlinear time-varying systems with sensor saturation so as to complement the excellent results in [16–19].

Motivated by the above discussion, in this paper, we aim to investigate the probability-guaranteed H_∞ finite-horizon filtering problem for a class of nonlinear discrete time-varying systems with sensor saturations. The considered uncertain parameters are governed by mutually independent stochastic variables that abide by uniform distributions over the known finite ranges. A parameter-box is sought for designing the time-varying H_∞ filter such that the H_∞ performance requirement is guaranteed with pre-specified probability constraints. A computational algorithm is presented to characterize the solution to the finite-horizon filtering problem based on the semi-definite programming method. A simulation example is given to show the effectiveness of the filtering scheme. The main contributions of this paper can be highlighted as follows: (1) the system model addressed is quite comprehensive that covers uncertain parameters, nonlinearities as well as sensor saturations, thereby better reflecting the reality; (2) the filtering problem addressed is dealt with over a finite-horizon with probability performance constraint; and (3) the algorithm developed is of recursive nature that is suitable for online applications.

The remainder of this paper is arranged as follows. Section 2 briefly introduces the problem under consideration. In Section 3, the probabilistic performance requirement is expressed as a set of linear matrix inequalities (LMIs) and the H_∞ performance analysis is conducted by means of solving a set of difference linear matrix inequalities (DLMIs) [20,21]. Moreover, a computational algorithm is presented to characterize the design of the probability-guaranteed robust H_∞ finite-horizon filter. An illustrative example is utilized in Section 4 to show the effectiveness of the proposed approach. The paper is concluded in Section 5.

Notations. The notations used throughout the paper are standard. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. For a matrix P, P^T and P^{-1} represent its transpose and inverse, respectively. The notation $P > 0$ ($P \geq 0$) means that matrix P is real, symmetric and positive definite (positive semi-definite). Prob$\{\cdot\}$ is used for the occurrence probability of the event "$\cdot$". $\|\cdot\|$ denotes the Euclidean norm of a vector. diag$\{\cdots\}$ stands for a block-diagonal matrix. $l_2[0, N-1]$ is the space of square summable vector-value functions on an interval $[0, N-1]$ with the norm $\|\nu\|_{[0,N-1]} = \sqrt{\sum_{k=0}^{N-1}\|\nu(k)\|^2}$. I and 0 represent the identity matrix and the zero matrix with appropriate dimensions, respectively. In symmetric block matrices or long matrix expressions, we use a star "$*$" to represent a term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem formulation and preliminaries

In this paper, we consider the following class of nonlinear uncertain time-varying systems defined on $k \in \{0, 1, \ldots, N-1\}$:

$$\begin{cases} x(k+1) = A^{(\alpha)}(k)x(k) + B^{(\alpha)}(k)f(x(k)) + D^{(\alpha)}(k)\omega(k) \\ y(k) = \sigma(C(k)x(k)) + E(k)\omega(k) \\ z(k) = M(k)x(k) \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $y(k) \in \mathbb{R}^m$ is the measured output, $z(k) \in \mathbb{R}^r$ is the output vector to be estimated, $\omega(k) \in \mathbb{R}^p$ is the disturbance input belonging to $l_2[0, N-1]$, $f(x(k))$ is the nonlinear function, the initial state $x(0)$ is an unknown vector, $\alpha = \begin{bmatrix}\alpha_1 & \alpha_2 & \cdots & \alpha_L\end{bmatrix}^T \in \mathbb{R}^L$is the uncertain parameter vector, and all α_i ($i = 1, 2, \ldots, L$) are assumed to be mutually independent random variables. Each α_i is uniformly distributed over $[\beta_i, \delta_i]$ where β_i and δ_i are known endpoints of $\alpha_i (i = 1, 2, \ldots, L)$. The uncertain parameter vector α lies in an L-dimensional hyper-rectangle $\mathbf{B}$ with the vertices set denoted by

$$V_{\mathbf{B}} = \left\{ \begin{bmatrix}\alpha_1 & \alpha_2 & \cdots & \alpha_L\end{bmatrix}^T \,\middle|\, \alpha_i \in \{\beta_i, \delta_i\}, i = 1, 2, \ldots, L \right\}. \tag{2}$$

Following [19], the uncertain matrices $A^{(\alpha)}(k)$, $B^{(\alpha)}(k)$ and $D^{(\alpha)}(k)$ in (1) are described by

$$\begin{aligned} A^{(\alpha)}(k) &= A_0(k) + \sum_{i=1}^{L}\alpha_i A_i(k), \\ B^{(\alpha)}(k) &= B_0(k) + \sum_{i=1}^{L}\alpha_i B_i(k), \\ C^{(\alpha)}(k) &= C_0(k) + \sum_{i=1}^{L}\alpha_i C_i(k). \end{aligned} \tag{3}$$

For a given sampling instant k, $A_i(k)$, $B_i(k)$, $D_i(k) (i = 1, 2, \ldots, L)$, $C(k)$, $E(k)$ and $M(k)$ are known constant matrices with appropriate dimensions. Accordingly, the uncertain matrices in (1) belong to the following general convex polytope

$$\Omega \triangleq \left\{ \left(A^{(\alpha)}(k), B^{(\alpha)}(k), D^{(\alpha)}(k)\right) \,\middle|\, \left(A^{(\alpha)}(k), B^{(\alpha)}(k), D^{(\alpha)}(k)\right) = \sum_{j=1}^{2^L} f_j \Omega^{(j)}(k),\ 0 \leq f_j \leq 1,\ \sum_{j=1}^{2^L} f_j = 1 \right\} \tag{4}$$

where $\Omega^{(j)}(k) = (A^{(j)}(k), B^{(j)}(k), D^{(j)}(k))$ $(j = 1, 2, \ldots, 2^L)$ are the vertex matrices. The relation between $\Omega^{(j)}(k)$ and $V_{\mathbf{B}}$ is given as follows:

$$A^{(j)}(k) = A(v_{\mathbf{B}}^{(j)}, k) = A(\alpha^{(j)}, k)$$

where $v_{\mathbf{B}}^{(j)}$ is the jth vertex of $\mathbf{B}$ generated by the parameter vector $\alpha^{(j)} = \begin{bmatrix}\alpha_1^{(j)} & \alpha_2^{(j)} & \cdots & \alpha_L^{(j)}\end{bmatrix}^T$, $\alpha_i^{(j)} \in \{\beta_i, \delta_i\}$ $(i = 1, 2, \ldots, L; j = 1, 2, \ldots, 2^L)$. $B^{(j)}(k)$ and $D^{(j)}(k)$ have the similar expressions.

The saturation function $\sigma(\cdot)$ is defined as

$$\sigma(v) = \begin{bmatrix}\sigma_1(v_1) & \sigma_2(v_2) & \cdots & \sigma_m(v_m)\end{bmatrix}^T \tag{5}$$

with $\sigma_i(v_i) = \mathrm{sign}(v_i)\min\{v_{i,\max}, |v_i|\}$, where $v_{i,\max}$ is the i-th element of the vector $v_{\max}$ with $v_{\max}$ being the saturation level.

To facilitate our development, we introduce the following definition.

Definition 1 (*[22]*)**.** A nonlinearity $\Phi(\cdot)$ is said to satisfy the sector-bounded condition if

$$(\Phi(v) - V_1 v)^T(\Phi(v) - V_2 v) \leq 0 \tag{6}$$

for some real matrices V_1, V_2, where $V = V_2 - V_1$ is a symmetric positive-definite matrix. In this case, we say that $\Phi(\cdot)$ belongs to the sector $[V_1, V_2]$.

Assumption 1. The nonlinear function $f(x(k))$ in (1) belongs to the sector $[U_1(k), U_2(k)]$ where $U_1(k)$ and $U_1(k)$ are real matrices of appropriate dimensions.

Noting that there exist the diagonal matrices K_1 and K_2 such that $0 \leq K_1 < I \leq K_2$, the saturation function $\sigma(C(k)x(k))$ in (1) can be written as

$$\sigma(C(k)x(k)) = K_1 C(k)x(k) + \Phi_y(C(k)x(k)) \tag{7}$$