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Optimal control modification for robust adaptive control with large adaptive gain

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ABSTRACT

In the presence of large uncertainties, a control system needs to be able to adapt rapidly to regain performance. Fast adaptation is referred to the implementation of adaptive control with a large adaptive gain so as to reduce the tracking error rapidly. However, a large adaptive gain can lead to high-frequency oscillations which can adversely affect robustness. A new adaptive law, called optimal control modification, is presented that can achieve robust adaptation with a large adaptive gain without incurring high-frequency oscillations. The modification is based on a minimization of the $\mathcal{L}_2$ norm of the tracking error bounded away from some lower bound, formulated as an optimal control problem. The optimality condition is used to derive the modification based on the Pontryagin's Minimum Principle. The optimal control modification is shown to improve robustness of the standard MRAC without significantly compromising the tracking performance. Flight control simulations demonstrate the effectiveness of the new adaptive law. A series of recent, successful flight tests of this adaptive law on a NASA F/A-18A aircraft at NASA Dryden Flight Research Center further demonstrate the effectiveness of the optimal control modification adaptive law.

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1. Introduction

Adaptive control is a potentially promising technology that can improve a control system performance over a conventional fixed-gain controller. In recent years, adaptive control has been receiving a significant amount of attention. In aerospace applications, adaptive control has been demonstrated in a number of flight vehicles. For example, NASA conducted in the last decade a flight test program of a neural net intelligent flight control system on board a modified F-15 test aircraft [1]. The ability to accommodate system uncertainties and to improve fault tolerance of a control system is a major selling point of adaptive control since conventional gain-scheduled or fixed-gain control methods are viewed as being less capable of handling off-nominal conditions caused by faults and or failures. Nonetheless, these conventional control methods tend to be robust to disturbances and unmodeled dynamics when operated as intended.

In spite of the advances made in the field of adaptive control, there exist several challenges related to the implementation of adaptive control technology in safety-critical systems. Verification and validation of adaptive control remains a major hurdle to overcome [2]. This hurdle can be traced to the lack of performance and stability metrics for adaptive control which poses a major design challenge for adaptive control. Stability robustness of adaptive control is also a major overriding concern. The fact that adaptive control is nonlinear certainly makes it inherently much more difficult to ascertain stability robustness.

Even as adaptive control has been used with limited success in some applications, the possibility of high-gain control due to fast adaptation can be an issue. In certain applications, fast adaptation is needed in order to improve the tracking performance rapidly when a system is degraded significantly due to a plant damage or failure that causes large changes in system dynamics. In these situations, a large adaptive gain can be used to reduce the tracking error rapidly. However, there typically exists a balance between stability and adaptation. Fast adaptation leads to an improved tracking performance, but by the same token can also result in poor robustness that could adversely affect stability of a control system [3]. Some recent adaptive control methods have addressed fast adaptation and high-gain control, such as the $\mathcal{L}_1$ adaptive control [4] and the hybrid direct–indirect adaptive control [5], as well as other techniques. In the $\mathcal{L}_1$ approach, the implementation of a low-pass filter on the adaptive control signal effectively suppresses any high frequency oscillations that may occur due to fast adaptation. In the limit, the $\mathcal{L}_1$ method provides a time delay margin bounded away from zero. In the hybrid approach, direct and indirect adaptive control are blended together within the same control architecture. The indirect adaptive law uses a recursive least-squares method to adjust parameters of a controller to reduce the modeling error, and the direct adaptive law then handles any residual tracking error using a smaller adaptive gain.

To increase stability robustness of MRAC due to fast adaptation, robust modification adaptive laws can also be used. Two well-known robust modification adaptive laws that have been used

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extensively in adaptive control are the σ modification [6] and ε modification [7]. Robust modification effectively introduces a damping mechanism into an adaptive law so as to prevent adaptive parameter bursting phenomena due to lack of robustness [3]. This paper introduces a new adaptive law based on an optimal control formulation that minimizes the $\mathscr{L}_2$ norm of the tracking error bounded away from some lower bound [8]. The new adaptive law, referred to as optimal control modification, can enable fast adaptation without loss of robustness. The analysis shows that the optimal control modification adaptive law can improve stability robustness of adaptive control to unmodeled dynamics. Simulations and flight testing have been conducted with this new adaptive law. The results support the effectiveness of the optimal control modification adaptive law.

2. Optimal control modification

Given a nonlinear plant as

$$\dot{x}(t) = Ax(t) + B[u(t) + f(x(t))] \tag{1}$$

where $x(t) : [0, \infty) \to \mathbb{R}^n$ is a state vector, $u(t) : [0, \infty) \to \mathbb{R}^p$ is a control vector, $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times p}$ are known such that the pair (A, B) is controllable, and $f(x(t)) : \mathbb{R}^n \to \mathbb{R}^p$ is a matched uncertainty.

Assumption. The uncertainty $f(x(t))$ can be linearly parametrized as

$$\begin{aligned} f(x(t)) &= \sum_{i=1}^{m} \theta_i^* \phi_i(x(t)) + \varepsilon(x(t)) \\ &= \Theta^{*\top} \Phi(x(t)) + \varepsilon(x(t)) \end{aligned} \tag{2}$$

where $\Theta^* \in \mathbb{R}^{m\times p}$ is an unknown constant ideal weight matrix that represents a parametric uncertainty, $\Phi(x(t)) : \mathbb{R}^n \to \mathbb{R}^m$ is a vector of known bounded basis functions or regressors that are continuous and differentiable in x, and $\varepsilon(x(t)) : \mathbb{R}^n \to \mathbb{R}^p$ is an approximation error.

The set of basis functions $\Phi(x(t))$ is chosen such that the approximation error $\varepsilon(x(t))$ is small on a compact domain $x(t) \in \mathbb{R}^n$. There are several function approximation methods that can be used for selecting suitable basis functions. For example, the universal approximation theorem for sigmoidal neural networks by Cybenko can be used for selecting sigmoidal basis functions $\Phi(x(t))$ [9]. Similarly, the Micchelli's theorem provides a theoretical basis for a neural net design using radial basis functions to keep the approximation error $\varepsilon(x(t))$ small [10]. Other function approximation methods such as Chebyshev orthogonal polynomials have also been used [11].

The feedback controller $u(t)$ is specified by

$$u(t) = -K_x x(t) + K_r r(t) - u_{ad}(t) \tag{3}$$

where $r(t) : [0, \infty) \to \mathbb{R}^p \in \mathscr{L}_\infty$ is a command vector, $K_x \in \mathbb{R}^{p\times n}$ is a stable gain matrix such that $A - BK_x$ is Hurwitz, $K_r \in \mathbb{R}^{p\times p}$ is a gain matrix for $r(t)$, and $u_{ad}(t) \in \mathbb{R}^p$ is a direct adaptive signal which estimates the parametric uncertainty in the plant such that

$$u_{ad}(t) = \Theta^\top(t) \Phi(x(t)) \tag{4}$$

where $\Theta(t) \in \mathbb{R}^{m\times p}$ is an estimate of the parametric uncertainty Θ^*.

Then, the reference model is specified as

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t) \tag{5}$$

where $A_m \in \mathbb{R}^{n\times n}$ and $B_m \in \mathbb{R}^{n\times p}$ are given by

$$A_m = A - BK_x \tag{6}$$

$$B_m = BK_r. \tag{7}$$

Let $\tilde{\Theta}(t) = \Theta(t) - \Theta^*$ be an estimation error of the parametric uncertainty and define the tracking error as $e(t) = x_m(t) - x(t)$, then the tracking error equation becomes

$$\dot{e}(t) = A_m e(t) + B\left[\tilde{\Theta}^\top(t) \Phi(x(t)) - \varepsilon(x(t))\right]. \tag{8}$$

Proposition. *The following optimal control modification adaptive law*

$$\dot{\Theta}(t) = -\Gamma \Phi(x(t)) \left[e^\top(t) P - \nu \Phi^\top(x(t)) \Theta(t) B^\top P A_m^{-1}\right] B \tag{9}$$

provides an update law that minimizes $\|e(t)\|_{\mathscr{L}_2}$ associated with an infinite-time horizon cost function

$$J = \lim_{t\to\infty} \frac{1}{2} \int_0^{t_f} [e(t) - \Delta(t)]^\top Q [e(t) - \Delta(t)]\, dt \tag{10}$$

subject to Eq. (8), where Δ represents the unknown lower bound of the tracking error, $Q = Q^\top > 0 \in \mathbb{R}^{n\times n}$ is a weighting matrix, $\Gamma = \Gamma^\top > 0 \in \mathbb{R}^{m\times m}$ is an adaptive gain matrix, $\nu > 0 \in \mathbb{R}$ is a modification parameter, and $P = P^\top > 0 \in \mathbb{R}^{n\times n}$ solves

$$PA_m + A_m^\top P = -Q. \tag{11}$$

Proof. The cost function J is convex and represents the distance measured from a point on the trajectory of $e(t)$ to the normal surface of a hypersphere $B_\Delta = \{e(t) \in \mathbb{R}^n : \|e(t)\| \le \|\Delta(t)\|\} \subset \mathscr{D} \subset \mathbb{R}^n$. The cost function is designed to provide stability robustness by not seeking an asymptotic tracking error that tends to zero as $t_f \to \infty$, but rather one that tends to some lower bound away from the origin. By not requiring an asymptotic tracking error, the adaptation can be made more robust. Therefore, the tracking performance can be traded with stability robustness by a suitable selection of the modification parameter ν. Increasing the tracking performance by reducing ν will decrease stability robustness of the adaptive law to unmodeled dynamics and vice versa. □

This optimal control problem can be formulated by the Pontryagin's Minimum Principle. Defining a Hamiltonian

$$\begin{aligned} H\left(e(t), \tilde{\Theta}(t)\right) &= \frac{1}{2}[e(t) - \Delta(t)]^\top Q [e(t) - \Delta(t)] \\ &+ p^\top(t)\left[A_m e(t) + B\tilde{\Theta}^\top(t) \Phi(x(t)) - B\varepsilon(x(t))\right] \end{aligned} \tag{12}$$

where $p(t) : [0, \infty) \to \mathbb{R}^n$ is an adjoint variable, then the necessary condition gives

$$\dot{p}(t) = -\nabla H_e^\top = -Q[e(t) - \Delta(t)] - A_m^\top p(t) \tag{13}$$

with the transversality condition $p\left(t_f \to \infty\right) = 0$ since $e(0)$ is known. Treating $\tilde{\Theta}^\top(t) \Phi(x)$ as a control variable, then the optimality condition is obtained by

$$\nabla H_{\tilde{\Theta}^\top \Phi} = p^\top(t) B. \tag{14}$$

The adaptive law can then be formulated by a gradient update law as

$$\begin{aligned} \dot{\tilde{\Theta}}(t) &= -\Gamma \nabla H_{\tilde{\Theta}^\top} = -\Gamma \Phi(x(t)) \nabla H_{\tilde{\Theta}^\top \Phi} \\ &= -\Gamma \Phi(x(t)) p^\top(t) B. \end{aligned} \tag{15}$$

An "approximate" solution of $p(t)$ can be obtained using a "sweep" method [12] by letting $p(t) = W(t) e(t) + R(t) \Theta^\top(t) \Phi(x(t))$ where $W(t) : [0, \infty) \to \mathbb{R}^{n\times n}$ and $R(t) :$