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LMI-based algorithm for strictly positive real systems with static output feedback

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ABSTRACT

An algorithm based on Linear Matrix Inequalities (LMIs) is proposed to find a constant output feedback matrix K_o and a constant output tandem matrix F such that the controlled system $\{A - BK_oC, B, FC\}$ is Strictly Positive Real (SPR). The number of output variables of the plant $\{A, B, C\}$ is greater than or equal to the number of its input variables. Considering that SPR systems with static output feedback are related to SPR systems with static state feedback, as shown in this manuscript, the first step of the algorithm is to find a matrix F such that all transmission zeros of the system $\{A, B, FC\}$ have negative real parts. After finding this matrix F, an output feedback matrix K_o such that the system $\{A - BK_oC, B, FC\}$ is SPR is found. Another algorithm is proposed to specify a decay rate. The results are applied to the simulation of electrical stimulation for paraplegic patients, to vary knee joint angle from 0° to 60°.

1. Introduction

A finite-dimensional linear time-invariant system that is Strictly Positive Real (SPR) has the following properties: it is a passive system, it is asymptotically stable and all its transmission zeros have negative real parts. Furthermore, negative feedback of a passive dynamic system is internally stable. There exist significant results about SPR systems stability, such as Popov's asymptotic hyperstability [1]. More recent results are given in [2,3]. These results have been applied, for instance, in the design of adaptive control systems [4–11], Variable Structure Control (VSC) systems [12–15], switched control of affine systems [16] and stabilization of output feedback uncertain systems [17–19]. The first step in these applications was to build a compensation structure to turn the system into an SPR system and, then, the control law was designed using the SPR stability results.

A problem related to this design method, called SPR synthesis, is the following: given a linear, time invariant, controllable and observable plant $\{A, B, C\}$, find a constant output feedback matrix K_o and a constant output tandem matrix F such that the controlled system $\{A - BK_oC, B, FC\}$ is SPR. In [10], it was shown that this problem is equivalent to an output feedback stabilization problem. When the plant has the number of inputs m equal to the number of outputs p, the necessary and sufficient condition for this problem is that all the transmission zeros of the plant must have negative real parts and $\det(CB) \neq 0$ [7,9,10].

In [6,14,15,20–22], it was studied the solution for the problem above using Linear Matrix Inequalities (LMIs) [23]. The advantage of this method is that LMIs, when feasible, can be easily solved, using available softwares [24,25]. This method also allows to consider other design specifications, such as plant uncertainties, decay rate, constraints in the output and in the input [14,15,26]. For the problem stated above, with $p > m$, only sufficient conditions based on LMIs are known [15].

In this manuscript, an algorithm is proposed to get an SPR system by static output feedback, when the plant has the number of outputs greater than the number of inputs. At this algorithm, the characteristic matrix A is expressed as $A = BA_B + B_\perp A_o$, where $B_\perp^T B = 0$ and $\operatorname{rank}([BB_\perp]) = n$. The first step is to find a state feedback matrix K_s and a tandem matrix F such that the static state feedback system $\{A - BK_s, B, FC\}$ is SPR. Then, from this solution, an SPR system $\{A - BK_oC, B, FC\}$ with an output feedback matrix K_o is found. Another algorithm is also proposed to specify a decay rate.

Output feedback is one of the most important questions in control engineering. Output feedback controllers are applied for plants where some state variables are not available for measurement and feedback, due to its simplicity and low cost. However, output feedback controllers are more difficult to design than state feedback controllers. For instance, as far as the authors knowledge, given a linear time-invariant system $\{A, B, C\}$, there exist only sufficient LMI-based conditions for the design of an output gain K_0 such that the controlled system $\{A + BK_0C, B, C\}$ is asymptotically stable. Various necessary and sufficient conditions

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for output feedback control design are available in the literature. But these conditions do not solve a large class of problems. The difficulties of static output feedback design are pointed in [27].

The notations used in this manuscript are: $\mathbb{R}^{m\times n}$, for the set of real matrices with m rows and n columns; A^T, for the transpose of the matrix A; $A > (\geq)0$, to indicate that the matrix A is positive (semi-)definite; z^* of G^*, for the conjugate of the complex of the number z of matrix G.

2. Statement of the problem

Consider the linear time-invariant plant, that is controllable and observable:

$$\dot{x} = Ax + Bu, \qquad y = Cx, \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control input, $y \in \mathbb{R}^p$, is the system output, $A \in \mathbb{R}^{n\times n}$ is the characteristic matrix of the system, $B \in \mathbb{R}^{n\times m}$ is the input matrix and $C \in \mathbb{R}^{p\times n}$ is the output matrix, with $\mathrm{rank}(C) = p$, $\mathrm{rank}(B) = \mathrm{rank}(CB) = m$ and $p \geq m$.

Definition 1 ([1]). The transfer matrix $G(s) \in \mathbb{R}^{m\times m}$ of the system (1) is Positive Real (PR) if the following conditions are satisfied:

(a) the elements of $G(s)$ do not have poles with positive real part;
(b) $G^*(s) = G^T(s^*)$, and
(c) the Hermitian matrix $J(s) = G(s) + G^T(s^*)$ is positive semidefinite in $Re(s) > 0$.

Definition 2 ([1]). The transfer matrix $G(s)$ is Strictly Positive Real (SPR) if $G(s-\epsilon)$ is PR for any $\epsilon > 0$.

In [1], SPR systems are related to Popov's asymptotic hyperstability. The necessary and sufficient condition for SPR systems is also presented in [1].

Lemma 1 ([1]). *Consider a controllable and observable system given by (1). The transfer matrix of the system $G(s) = C(sI-A)^{-1}B$ is SPR if and only if there exists a matrix $P = P^T$, such that:*

$$PA + A^TP < 0, \qquad B^TP = C, \quad P > 0. \tag{2}$$

Theorem 1 ([9,10]). *Consider the system in Fig. 1. Then, there exists a constant matrix K_z, such that the static output feedback system is SPR if and only if the following conditions hold:*

(i) $C_zB_z = (C_zB_z)^T > 0$;
(ii) *all transmission zeros of the plant $\{A_z, B_z, C_z\}$ have negative real parts.*

As defined in [28], the transmission zeros of a system whose number of outputs p is greater than or equal to the number of inputs m are the values of $s \in \mathbb{C}$ such that:

$$\mathrm{rank}\begin{bmatrix} sI-A & B \\ -C & 0 \end{bmatrix} < n+m.$$

Based on Lemma 1 and Theorem 1, the following problems were proposed.

Problem 1. Given the plant $\{A, B, C\}$ defined in (1), find LMI-based conditions, for the existence of constant matrices $F \in \mathbb{R}^{m\times p}$ and $K_s \in \mathbb{R}^{m\times n}$, such that the static state feedback system described in Fig. 2 is SPR.

Problem 2. Given the plant $\{A, B, C\}$ defined in (1), find LMI-based conditions, using LMIs, for the existence of constant matrices F and $K_o \in \mathbb{R}^{m\times p}$, such that the static output feedback system described in Fig. 3 is SPR.

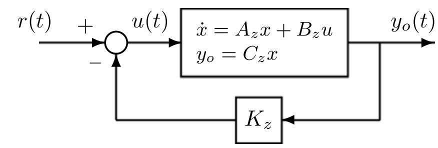

Fig. 1. Output feedback system of Theorem 1.

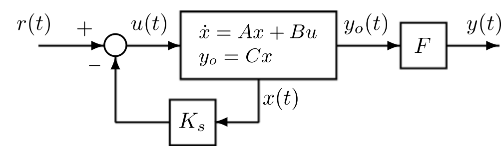

Fig. 2. State feedback system of Problem 1.

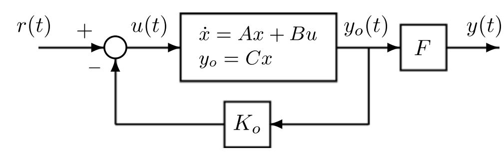

Fig. 3. Output feedback system of Problem 2.

Theorem 2. *There exists a solution for Problem 2 if and only if Problem 1 has a solution.*

Proof (*Sufficiency*). Suppose that there exists a solution for Problem 1. Thus, there exist matrices K_s and F such that the system $\{A-BK_s, B, FC\}$ is SPR. Therefore, considering in Fig. 1, $A_z = A-BK_s$, $B_z = B$ and $C_z = FC$, the static output feedback system is SPR for $K_z = 0$. Thus, according to Theorem 1, the following conditions hold:

(i) $FCB = (FCB)^T > 0$;
(ii) all transmission zeros of the system $\{A-BK_s, B, FC\}$ have negative real parts.

The transmission zeros of a plant $\{A, B, FC\}$ are the numbers $s \in \mathbb{C}$, such that:

$$\mathrm{rank}\begin{bmatrix} sI-A & B \\ -FC & 0 \end{bmatrix} < n+m.$$

By elementary operations in the columns of the matrix that defines the transmission zeros of $\{A-BK_s, B, FC\}$, one can observe that these transmission zeros are the same transmission zeros of $\{A, B, FC\}$:

$$\mathrm{rank}\begin{bmatrix} sI-A+BK_s & B \\ -FC & 0 \end{bmatrix} = \mathrm{rank}\begin{bmatrix} sI-A & B \\ -FC & 0 \end{bmatrix} < n+m.$$

Again, from Theorem 1, now for $A_z = A$, $B_z = B$ and $C_z = FC$ in Fig. 1, considering that $FCB = (FCB)^T > 0$ and all transmission zeros of the plant $\{A, B, FC\}$ present negative real parts, then there exists an output feedback matrix K_z, such that the system $\{A - BK_zFC, B, FC\}$ is SPR. Thus, the matrices F and $K_o = K_zF$ correspond to a solution for Problem 2.

(Necessity) Consider the existence of matrices K_o and F, such that the static output feedback system $\{A-BK_oC, B, FC\}$ is SPR and define $K_s = K_oC$. Then, there exist matrices K_s and F, such that the static state feedback system $\{A-BK_s, B, FC\}$ is SPR. □

Based on Lemma 1 and Theorem 2, an algorithm for the solution of Problem 2 is proposed. This algorithm is described in the next section.