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A geometric approach to the general autonomous regulator problem in the time-delay framework

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ABSTRACT

The aim of this paper is to show the applicability of geometric techniques to a regulation problem for linear, time-delay systems. Given a plant whose dynamics equations include delays and an exosystem that generates a reference signal, the problem we consider consists in finding a feedback regulator which guarantees asymptotic stability of the regulation loop and asymptotic command following of the reference signal, for any initial condition of the overall system in the presence of disturbances. By associating to the time-delay plant a corresponding abstract system with coefficients in a ring, it is possible to place our investigation in a finite dimensional algebraic context, where intuition and results obtained in the classical case, that is without delays, may be exploited. In particular, using tools and methods of the geometric approach to systems with coefficients in a ring, sufficient conditions for the solvability of the considered problem are found and a constructive procedure, which works under specific hypotheses, is given.

1. Introduction

Time-delay dynamical systems of various kinds appear frequently in industrial applications, where delays are unavoidable effects of the transportation of materials, as well as in control applications where information is dispatched along slow or very long communication lines, like in tele-operated systems, networked systems, large integrated communication control systems. The study of control problems concerning time-delay systems has, for that reason, attracted the attention of several authors and motivated, in the last years, large research efforts (see the Proceedings of the IFAC Workshop on Time-delay Systems [1] and the books [2,3] for an account of the most recent literature).

Among the various approaches developed for dealing with time-delay systems and related control problems, the one based on the use of geometric methods, in the spirit of [4,5], has proved to be particularly effective in many situations, as shown in [6,7] and in the references therein. Application of geometric methods to time-delay systems relies on the possibility of associating to any linear, time-delay system an abstract system with coefficients in a suitable ring. In this way, control problems arising in the time-delay framework can be equivalently formulated, and possibly solved, in an algebraic framework where input/output behaviors have finite-dimensional state space realizations and geometric properties can be workably defined and characterized.

Here, we apply this approach to the study of the so called Multivariable Autonomous Regulator Problem for linear, time-delay systems. The problem has already been considered and solved for classical linear systems without delay, namely linear time-invariant systems with coefficients in the real field, in [8] using an algebraic approach and in [9] using a geometric approach (see also [10]). However, to the best of the authors' knowledge, no solutions of the Multivariable Autonomous Regulator problem for time-delay systems or, more generally, for systems with coefficients over a ring, can be found in the available literature. The importance of time-delay systems in engineering applications motivates this work, where the geometric approach combined with methods of algebra and ring theory provides the theoretical bases and the computational procedure to deal with the problem at issue.

Briefly, for a given linear, time-delay plant Σ_{pd}, subject to a disturbance input, the Multivariable Autonomous Regulator Problem, which is formally stated in Section 3, consists in finding a feedback regulator Σ_{rd} which guarantees asymptotic stability of the regulation loop and, for any initial condition, asymptotic tracking of the reference signal generated by an exosystem Σ_e. After recalling, in Section 2, the formal relationship between time-delay systems and systems with coefficients in a ring and reviewing some basic notions and results of the geometric approach, the problem is restated, in an equivalent form, in geometric terms for systems over rings in Section 4. This procedure

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can be viewed as a change of domain for the problem at issue: from the framework of time-delay systems to the framework of systems with coefficients in a ring. Then, following the same conceptual approach as in [10], we find, in Section 5, sufficient conditions for solvability of the Multivariable Autonomous Regulator Problem in the ring framework. Going back to the original domain and re-interpreting such conditions in the time-delay framework, we can apply the result we have found to the original problem. In Section 6, a procedure for constructing a solution, based on geometric and algebraic computations, is presented.

Besides providing a practical way to deal with the Multivariable Autonomous Regulator Problem for linear, time-delay systems, the results obtained show once more the applicability and the efficacy of the geometric approach combined with methods of algebra and ring theory in solving control problems for time-delay systems.

An earlier version of the results of this paper, in the simpler situation where disturbances on the plant were not considered, was given in [11].

2. Preliminary results

Let us consider a linear, time-invariant, time-delay system Σ_d defined by equations of the form

$$\Sigma_d = \begin{cases} \dot{x}(t) = \sum_{i=0}^{a} A_i x(t - ih) + \sum_{i=0}^{b} B_i u(t - ih), \\ y(t) = \sum_{i=0}^{c} C_i x(t - ih), \end{cases} \tag{1}$$

where, denoting by $\mathbb{R}$ the field of real numbers, $x(\cdot)$ belongs to the state space $\mathbb{R}^n$, $u(\cdot)$ belongs to the input space $\mathbb{R}^m$, $y(\cdot)$ belongs to the output space $\mathbb{R}^p$, A_i, with $i = 0, 1, \ldots, a$, B_i, with $i = 0, 1, \ldots, b$, C_i, with $i = 0, 1, \ldots, c$, are matrices of suitable dimensions with entries in $\mathbb{R}$, and $h \in \mathbb{R}^+$ is a given time-delay.

By introducing the delay operator δ, defined, for any function $f(t)$, by $\delta f(t) = f(t - h)$, we can rewrite Eqs. (1) as

$$\begin{cases} \dot{x}(t) = \sum_{i=0}^{a} A_i \delta^i x(t) + \sum_{i=0}^{b} B_i \delta^i u(t), \\ y(t) = \sum_{i=0}^{c} C_i \delta^i x(t). \end{cases} \tag{2}$$

Then, by formally substituting the operator δ by the indeterminate Δ, we can consider the matrices

$$A = \sum_{i=0}^{a} A_i \Delta^i, \qquad B = \sum_{i=0}^{b} B_i \Delta^i, \qquad C = \sum_{i=0}^{c} C_i \Delta^i,$$

having their elements in the ring $\mathbb{R}[\Delta]$ of the polynomials in the indeterminate Δ with coefficients in the field $\mathbb{R}$.

Many authors [12–17] have approached the study of time-delay systems of the form (1), by analyzing the algebraic properties of the associated matrices A, B and C. Taking a step further, by the procedure introduced and discussed in [6,17], we can associate to the system Σ_d the system Σ, defined over the ring $\mathbb{R}[\Delta]$ by the set of equations

$$\Sigma = \begin{cases} x(t+1) = A x(t) + B u(t), \\ y(t) = C x(t). \end{cases} \tag{3}$$

With a slight abuse of notation, in (3), we denote by $x(\cdot)$ an element of the free state module $X = (\mathbb{R}[\Delta])^n$, by $u(\cdot)$ an element of the free input module $U = (\mathbb{R}[\Delta])^m$, and by $y(\cdot)$ an element of the free output module $Y = (\mathbb{R}[\Delta])^p$.

The system Σ, derived as described above, is an exemplification of system with coefficients in a ring. Abstract systems with coefficients in a ring R have been considered in the literature in connection with different problems and applications (see, e.g., [18,19] and the references therein). Their study has, in general, provided a better insight into the properties of classical dynamical systems with coefficients in the real field $\mathbb{R}$, which in particular is a ring, and it has been useful in dealing with systems with integer coefficients, time-delay systems and families of parameter depending systems.

Note that Σ_d and Σ are quite different objects from a dynamical point of view. However, they share the structural properties that depend on the defining matrices and, in particular, they have the same signal flow graph. This fact implies that control problems concerning the input/output behavior of Σ_d can be formulated naturally in terms of the input/output behavior of Σ. Solutions found in the framework of systems over rings can often be interpreted in the original time-delay framework, providing in this way a solution to the problem at issue. The advantage of working in the ring framework consists in the possibility of employing algebraic tools and of dealing with finite dimensional modules instead of infinite dimensional vector spaces, like the state space of Σ_d (see [17] for comments on this point). In particular, using the associated systems with coefficients in a ring, it is possible to extend to time-delay systems the methods and tools of the geometric approach (see [6] for an account of the geometric approach for systems with coefficients in a ring).

Remark 1. It is worth noting that using models of the form (1) we assume that delays are commensurable: namely, that all the time-delays on the system variables are multiple of an elementary delay h. Although this assumption introduces an obvious restriction from the theoretical point of view, its impact on the modeling of engineering problems can be rendered negligible, provided that the delay h be chosen sufficiently small and, in particular, of the same order of magnitude of the accuracy guaranteed by computational tools in practical implementation. This fact makes the commensurable delays case important on its own and gives a first motivation for our choice.

On the other hand, for applying the approach described above to systems with non commensurable delays, one should deal with matrices having their elements in the ring $R = \mathbb{R}[\Delta_1, \Delta_2, \ldots, \Delta_k]$ of polynomials in several indeterminates with real coefficients. The algebra of such matrices is definitely more complicated then that of matrices with elements in the principal ideal domain $\mathbb{R}[\Delta]$, due to the fact that in $\mathbb{R}[\Delta_1, \Delta_2, \ldots, \Delta_k]$ not all ideals are principal, that is not all ideals have a single generator. In particular, this limits the applicability of geometric methods to our problem (see the discussion in the last part of this section and Proposition 1) and prevents a straightforward extension of our results.

Since a ring cannot, in general, be endowed with a natural metric structure, when using systems over rings, stability must be defined in a formal way. To this aim, let us introduce the set of polynomials $\mathcal{H}$ defined as follows:

$$\mathcal{H} = \{p(z, \Delta) \in R[z], \text{ such that } p(\gamma, e^{-\gamma h}) \neq 0 \text{ for all } \gamma \in \mathbb{C}, \text{ with } \operatorname{Re}\gamma \geq 0\},$$

where $\mathbb{C}$ denotes the field of complex numbers.

The set $\mathcal{H}$ is said to be a Hurwitz set of polynomials and a system Σ of the form (3) with coefficients in R is said $\mathcal{H}$-stable if and only if $\det(zI - A)$ belongs to $\mathcal{H}$ (see [16,20]). The importance of this notion is due to the obvious fact that stability of the time-delay system Σ_d in the differential framework corresponds to $\mathcal{H}$-stability of the associated system Σ in the ring framework.

The basic notions of the geometric approach we will need in the following are briefly recalled below.