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Optimal geometric motion planning for a spin-stabilized spacecraft

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ABSTRACT

A method requiring low-computational overhead is presented which generates low-torque reference motions between arbitrary orientations for a spin-stabilized spacecraft. The initial stage solves a constrained optimal control problem deriving analytical solutions for a class of smooth and feasible reference motions. Specifically, for a quadratic cost function an application of Pontryagin's maximum principle leads to a completely integrable Hamiltonian system that is, exactly solvable in closed-form, expressed in terms of several free parameters. This is shown to reduce the complexity of a practical motion planning problem from a constrained functional optimization problem to an unconstrained parameter optimization problem. The generated reference motions are then tracked using an augmented quaternion feedback law, consisting of the sum of a proportional plus derivative term and a term to compensate nonlinear dynamics. The method is illustrated with an application to re-point a spin-stabilized agile micro-spacecraft using zero propellant. The low computational overhead of the method enhances its suitability for on-board motion generation.

1. Introduction

Spin stabilization and three-axis stabilization are methods used to maintain the pointing direction of a spacecraft. With spin stabilization the entire spacecraft rotates around its own pointing axis using the gyroscopic effect [1]. Advantages of spin stabilization are that it is a simple and passive way of keeping the spacecraft pointing in a certain direction. Designers of early satellites used spin-stabilization and examples include NASA's Pioneer 10 and 11 spacecraft, the Lunar Prospector, and the Galileo Jupiter orbiter. The dynamics and control of a spin stabilized spacecraft, under varying mission assumptions, have been extensively researched; see for example [2–5]. Such passive control is particularly attractive to a limited resource spacecraft which can switch off their active GNC system during passive stabilization and use the battery power to perform operational tasks. However, a disadvantage of passive stabilization is that it cannot be used to accurately re-point the antennas or optical instruments. Therefore, missions that require frequent re-pointing tend to use three-axis stabilization. Three-axis stabilization can be achieved with gas jet actuators or reaction wheels using classical attitude control methods such as proportional plus derivative (PD) type controllers, based on quaternion-feedback for large angle maneuvers [6] or eigenaxis rotations [7].

The development of nano and micro-spacecraft (from the 1 kg Cubesat to the more enhanced micro-spacecraft of around 150 kg) are cost effective alternatives to a conventional spacecraft that pose new engineering challenges. One such challenge is that their small reaction wheels can generate a very limited torque for active control. For example Surrey Satellite Technology Limited (SSTL) is now developing a reaction wheel with a maximum torque capability of 0.1 N m, for future use on micro-satellites (100–150 kg). Therefore, in order to design propellant free maneuvers for a micro-spacecraft the reference motions have to be trackable within 0.1 N m. Once the desired pointing direction has been obtained, the spacecraft should use passive control to save battery power. Therefore, for a micro spacecraft it is desirable to use spin stabilization to maintain a pointing direction and perform small-torque motions (feasible with reaction wheels) for re-pointing. This type of maneuver could be achieved by "de-spinning" the pointing axis, performing a conventional rest-to-rest eigenaxis rotation and finally "spinning-up" around the pointing axis for passive stabilization. However, this three stage manoeuver is a highly inefficient procedure and in general would not be feasible with small torque.

This paper proposes a method requiring low-computational power for generating low-torque reference motions for a spacecraft constrained to spin about a single axis. By closely tracking this reference motion the spacecraft will minimize the "spin-up" or "de-spin" of the pointing axis providing gyroscopic stiffness and using small accumulated torque. The design of such low-torque reference motions subject to constraints are often formulated and solved in the context of constrained optimal control problems.

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These type of problems generally define the attitude kinematics and the Euler equations as equality constraints, with the performance index a function of control torques and/or time subject to prescribed boundary conditions and inequality constraints such as bounding the instantaneous torque [8–19]. In particular, designing minimum accumulated torque motions using pseudo-spectral direct transcription methods has proved instrumental in saving the International Space Station large amounts of precious propellant [20]. This method can however be computationally demanding. Other optimization approaches such as the locally optimal Euler Lagrange approach of calculus of variations were used to solve problems such as time-optimal attitude control [21] and the minimum fuel problem for a fixed time horizon [22]. These local methods have the added complexity of numerically solving two point boundary value problems onboard the satellite. The Hamilton–Jacobi–Bellman (HJB) approach from dynamic programming is globally stabilizing, but is numerically intractable. Solving it offline is possible numerically [23], but the global optimal solution is only approximated up to a certain order, which cannot be too high for practical implementation. For attitude control applications, these numerical optimization techniques are therefore still traditionally avoided onboard small micro-satellites with limited computational resources, such as those developed by SSTL.

The method in this paper uses geometric control theory [24–26] to reduce the computational expense of the optimization procedure. Geometric control theory has been used to derive coordinate free necessary and sufficient conditions for attitude controllability using gas jet actuators and momentum exchange devices. Furthermore, these results were used to design coordinate free control laws that stabilize the system around an equilibrium state [27]. Given the controllability of the system it is then desirable to plan a feasible and if possible optimal motion to track. Motion planning for systems defined on Lie groups, e.g. in this case the Special Orthogonal Group $SO(3)$, has been tackled using parameterizations of the manifold and using averaging to design small re-orientations [28]. However, this method is inadequate to perform large slew maneuvers and a global approach is required. A coordinate free formulation of an optimal control problem was proposed for the attitude motion planning of a spacecraft in Spindler [29]. For a trivial case of constant optimal angular velocities exact solutions for the corresponding rotations were derived. However, for time-dependent optimal angular velocities the method in [29] reverts to using numerical shooting methods, requiring numerical integration, in order to match the prescribed final orientation. Furthermore, this motion planning problem did not take torque requirement into consideration when planning motions.

In this paper, a method is proposed that can rapidly generate low-torque reference motions with low-computational overhead enabled through the machinery of geometric mechanics and control. Although the application of geometric control theory to solve left-invariant optimal control problems and their reduction to quadratures on three-dimensional Lie groups is well known [24,25], the particular problem considered here is solvable in closed form. This approach has the advantage that, for a spacecraft constrained to spin about one axis, the necessary conditions for optimality are guaranteed exactly with respect to a quadratic cost function. The analytic form of the solution then enables the construction of a practical cost function analytically which, in turn, requires the optimization of only a few free parameters. Specifically, Pontryagin's maximum principle enables the original motion planning problem to be reformulated as an unconstrained parameter optimization problem, thus, greatly reducing the computational demand. Moreover, a numerical optimization of a few parameters is used to match a prescribed final orientation while minimizing torque during the manoeuver. An augmented quaternion feedback is then used to track the designed reference motions.

This paper is presented as follows. In Section 2, the kinematic and dynamic models used are introduced. Although the equations for the rigid body are well known they are included here as the kinematics are expressed as quaternions and equivalently in the less conventional matrix form on the Special Unitary Group $SU(2)$. The quaternion differential equations are better suited for numerical integration as they do not have singularities and are conventionally used on-board spacecraft. Furthermore, the compact kinematic formulation on $SU(2)$ is also used as they enable the most natural and elegant analytical treatment in the derivation of the optimal motions. In Section 3, the derivation of a general form for the optimal motions in terms of several free parameters is given via Pontryagin's maximum principle and Lax pair integration [24–26]. The method assigns a simple quadratic cost function to the kinematic equations that constrain the rotational motion of the spinning axis about the pointing direction. It is then shown that the optimal angular velocity inputs are simple sinusoidal functions. This result viewed independently does not explicitly offer any new insight into the control problem as sinusoidal control of nonholonomic systems has already been covered extensively [30]. However, as the sinusoidal velocity inputs are derived in the setting of geometric mechanics and control it allows the corresponding motion to be derived completely analytically using Lax Pair integration [24,25]. The derived closed-form analytic expressions then enable the rapid generation of feasible reference motions. In Section 4, an unconstrained numerical parameter optimization method is undertaken to optimize the available parameters of the analytic solution to match the prescribed end-points and minimize accumulated torque amongst this subset of admissible motions.

In Section 5, a method is proposed to track the generated reference motions and a proof of the closed-loop stability is given in Appendix. Finally, an example is given which shows the reference motions to be feasible by the tracking controller, which is applied to perform re-pointing maneuvers for a spin-stabilized agile earth observation SSTL micro-satellite.

2. Models

The equations describing the attitude control problem are that of an asymmetric rigid body with external forces describing the effect of the reaction wheel torques. These consist of kinematic equations relating the angular position to the angular velocity, and the dynamic equations describing the evolution of angular velocity or, equivalently, angular momentum.

2.1. Kinematic equations

To obtain a global description of the problem the angular position is usually denoted by a rotation matrix in the Special Orthogonal Group $SO(3)$. In addition the angular position maybe described locally by parameterizing the rotation matrix using Euler angles [27]. However, in this paper both a quaternion representation and an equivalent matrix formulation on the Special Unitary Group $SU(2)$ is used ($SU(2)$ is isomorphic to the unit quaternions [25]). Again this representation is used as the quaternion differential equations are conventionally used on board spacecraft and are suited to numerical integration as they do not possess singularities. In addition the formulation on $SU(2)$ is used as it is the most natural and convenient for the analytical calculations. In terms of the angular velocity $\Omega_1, \Omega_2, \Omega_3$ in the body frame the quaternion representation of kinematics are taken to be:

$$\frac{d\bar{q}}{dt} = \frac{1}{2}\hat{\Omega}\bar{q} \tag{1}$$