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Shock capturing with entropy-based artificial viscosity for staggered grid discontinuous spectral element method

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ABSTRACT

This paper presents a shock capturing technique for a staggered grid discontinuous spectral element method (DSEM), which adds localized and smooth artificial viscosity to systems of nonlinear conservation laws. The artificial diffusivity model used in this work is a modified form of the entropy viscosity (EV) presented by Guermond et al. (2011). We extend the application of this method to high-order discontinuous schemes for the simulation of high speed flows with discontinuities on staggered grids. Direct implementation of the entropy viscosity method in DSEM leads to a non-smooth artificial viscosity, which in turn leads to oscillations and instability of the solution. To smoothen the artificial viscosity, the EV method is coupled with a spectral filter and an interface treatment technique. The resulting artificial viscosity is locally large near discontinuities and transitions smoothly to zero in smooth flow regions. The method enables using elements with orders higher than unity while avoiding adaptive mesh refinement and preserving the locality and compactness of the discontinuous Galerkin (DG) scheme. The method is implemented for the inviscid compressible Euler equations in two space dimensions and its effectiveness is demonstrated through its application to a series of benchmark problems.

1. Introduction

In the past decade, discontinuous Galerkin (DG) methods have increasingly become a viable alternative in the field of computational fluid dynamics (CFD). DG methods are a promising candidate to find high-fidelity solutions to the Euler equations that govern the gas dynamics in complex geometries. DG combins the properties of high-order discretization of the Galerkin finite element method and the local conservation typical of the finite volume method. Furthermore, the discretization offers advantages in terms of local mesh adaptation and efficient parallelization [1–9].

The staggered grid discontinuous spectral element method (DSEM) [1–3] is a notable member of the family of higher order methods. DSEM approximates the solution variables of conservation laws through a high-order local basis function in non-overlapping elements that may be oriented arbitrarily within an unstructured grid. The local, non-overlapping nature of the elements not only enables meshing of flow geometries of any complexity, it also ensures a high parallel efficiency and easy boundary condition implementation. DSEM has very small diffusion and dispersion errors and is spectrally convergent if the solution is smooth [1,2]. However, the implementation of DSEM for flows with discontinuities, such as supersonic flows, has proven quite challenging. Specifically, higher order approximations of discontinuous shock solutions that appear if the flow is supersonic, are troubled by Gibbs oscillations [4]. In this paper we focus on the application of DSEM to capture discontinuities in supersonic flows.

There has been an abundance of work on the extension of classical shock-capturing methodologies to high-order methods over the past two decades. They can be broadly categorized as the use of limiters [10], the weighting of stencils such as essentially non-oscillatory (ENO) and weighted essentially non-oscillatory (WENO) schemes [11,12] and the addition of artificial viscosity in the vicinity of shocks. Each of these methods is based on the smoothing of the solution local to the shock with the objective to essentially or entirely removing Gibbs oscillations.

Despite their indisputable success, the use of limiters in high-order DSEM approximations is challenging. Moreover, the use of limiters tends to drastically reduce the accuracy in a wide region near the shock.

Similarly, the extension of non-oscillatory numerical algorithms to complex geometries is complicated [13–15]. These methods reconstruct high-order approximations of the solution away from the discontinuities while resolving sharp profiles using additional

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degrees of freedom (DOF). However, they usually require carefully chosen reconstruction and numerical fluxes and their computational overhead for high-order approximations is high. Extension to multiple dimensions is limited to structured meshes.

Another approach to stabilize high-order numerical schemes is by adding diffusion to remove non-physical oscillations near shocks. Although this enables the capturing of the shocks and discontinuities, the diffusion near the sharp discontinuities may easily be excessive and tends to grow over time. Moreover, since viscosity is applied over the entire domain, one may lose the benefit of high order resolution. A method capable of adding viscosity which is localized in regions of shock, limited at contact discontinuities and virtually zero in smooth regions is desirable.

Von Neumann and Richtmyer [16] introduced the idea of a nonlinear artificial viscosity. This concept was adopted later by Baldwin and MacCormack [17] to simulate the interaction of shock waves with turbulent boundary layer. Later Jameson et al. [18] used this approach in the context of finite volume in combination with a Runge–Kutta time stepping scheme to simulate compressible flows in complex geometries. Adding artificial viscosity has long been the preferred method of shock capturing in the context of streamwise upwind Petrov–Galerkin (SUPG) finite element methods, as proposed by Hughes et al. [19]. Researchers such as Hartmann [20] and Aliabadi et al. [21] have used artificial viscosity for shock capturing with DG, albeit only for polynomial order $\mathcal{P} = 1$ solutions. Although this method, and other similar methods, are capable of adding the required amount of viscosity near shocks to spread the discontinuity and also control the numerical oscillations behind the shock wave, they can in turn overly diffuse the waves and produce incorrect wave speeds.

There have been other attempts to particularly design algorithms to add artificial viscosity to high-order discontinuous Galerkin methods. Persson and Peraire [22] introduced a polynomial order dependent artificial viscosity to produce sub-cell shock resolution for discontinuous Galerkin schemes. To locate the shocks in the flow field, Persson and Peraire developed a sensor based on the magnitude of the highest-order coefficients in an orthonormal representation of the solution. Later, Barter and Darmofal [23] used a reaction–diffusion equation to obtain a viscosity that is smooth in both time and space. Reinser et al. [24] used a similar reaction–diffusion equation in combination with the gradient based classical artificial viscosity to achieve a space–time smooth viscosity. Unfortunately, to achieve sufficient smoothing of the viscosity, one needs to choose a large diffusivity coefficient, which results in a stiff system of ODEs for time integration. The resulting time step restriction can be handled through implicit time integration. Implicit time stepping, however, not only is more laborious to implement, it also is more expensive per time step, and less parallel efficient for parallel computing.

Recently, Guermond et al. [25] proposed an artificial viscosity method for spectral methods, based on the local rate of entropy generation. In this method, the magnitude of the artificial viscosity is coupled to the magnitude of entropy generation. Since shocks generate entropy, the magnitude of entropy generation not only identifies the shock, but also is an excellent measure for the magnitude of the artificial viscosity. Hence, the addition of a numerical dissipation term proportional to the local entropy production rate contributes a numerical dissipation to the shock regions and hence removing Gibbs oscillations, while virtually no dissipation is added in the regions far from the shock to ensure an accurate computation of small-scale, turbulent flow away from shocks. In [25] the EV method was shown to be stable and accurate in combination with a continuous spectral element method. Zingan et al. [26] extended the use of EV to discontinuous finite element method, which is limited to the use of relatively low-order elements.

This work explores the feasibility of the entropy viscosity method within the framework of staggered grid DSEM. The standard entropy viscosity model was not designed for implementation in conjunction with a discontinuous collocation method. Specifically, element-to-element jumps in artificial viscosity leads to oscillations in solution gradients which can corrupt the smoothness and accuracy of the downstream flow field. We develop a smoother artificial viscosity by employing an elemental level filter and an interface treatment technique that results in a space–time smooth artificial viscosity. This artificially added coefficient is large and localized near discontinuities and transitions smoothly to zero in smooth flow regions. The method works in conjunction with high-order elements, avoiding adaptive mesh refinement, and preserves the locality and compactness of the DG scheme.

In Section 2, we briefly review the DSEM formulation and methodology. In Section 3, the implementation of EV in DSEM is explained. Section 4 explores the performance of DSEM-EV in dynamically capturing the discontinuous solution in one-dimensional and two-dimensional compressible flows. The final section is reserved for conclusions and recommendations for future work.

2. Formulation and methodology

Here, we briefly summarize the governing conservation laws and the collocated, staggered-grid DSEM formulation. For a detailed description of the DSEM and its computation for smooth flows in complex geometries, we refer to papers by Kopriva [1,2], Black [27] and Jacobs et al. [28].

2.1. Governing equations

For a compressible and Newtonian fluid that is assumed to obey the ideal gas equation of state, the compressible Navier–Stokes equations are given by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{1}$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u}\mathbf{u} + p\underline{\delta}) - \nabla \cdot \underline{\tau} = 0 \tag{2}$$

$$\frac{\partial \rho e}{\partial t} + \nabla \cdot (\rho e \mathbf{u} + p\underline{\delta} \cdot \mathbf{u}) - \nabla \cdot (\underline{\tau} \cdot \mathbf{u} + \kappa \nabla T) = 0 \tag{3}$$

where ρ is the density; $\mathbf{u}$ is the velocity vector; p is the static pressure; e is the internal energy; T is the temperature; κ is the thermal conductivity and $\underline{\delta}$ is the Kronecker tensor. The viscous stress tensor is

$$\underline{\tau} = \mu(2\underline{S}) + \left(\beta - \frac{2}{3}\mu\right)(\nabla \cdot \mathbf{u})\underline{\delta} \tag{4}$$

where μ is the dynamic viscosity; β is the bulk viscosity and $\underline{S}$ is the strain rate tensor,

$$\underline{S} = \frac{1}{2}[\nabla \mathbf{u} + (\nabla \mathbf{u})^T] \tag{5}$$

In this work the physical bulk viscosity of the fluid is taken to be zero following the Stokes hypothesis.

The governing equations can be non-dimensionalized with reference variables, and lead to the following non-dimensional Reynolds number, Re_f, Prandtl number, Pr_f, and Mach number, M_f as

$$Re_f = U_f L_f \rho_f / \mu, \quad Pr_f = c_p \mu / \kappa, \quad M_f = U_f / \sqrt{\gamma R T_f}$$

where subscript f denotes the reference scale; c_p is the constant-pressure specific heat; R is the gas constant and γ is the ratio of specific heats. Using these parameters, and assuming constant fluid properties, the non-dimensional form of Eqs. 1, 1 and 3 reads,