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Modeling of III-D problems of gas dynamics on multiprocessing computers and GPU

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ABSTRACT

The paper deals with a parallel algorithm for calculations on multiprocessor computers and GPU accelerators. The calculations of shock waves interaction with low density bubble results are presented [1]. This algorithm combines a possibility to capture a high resolution of shock waves, the second-order accuracy for TVD schemes, and a possibility to observe a low-level diffusion of the advection scheme.

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1. Introduction

The hyperbolic conservation laws and Euler equation of compressible fluid dynamics have been a subject of numerous researches for several decades, for a good reason. They are used in airplane and automobile design, in description of galaxy formations and supernova explosions, in studies of weather prediction and so on. Three-dimensional high-resolution schemes provide an opportunity to explore the behavior of waves of high intensity, resulting in the solution of various problems of unsteady gas dynamics.

Current problems of mathematical modeling in a realistic setting require increase of the size of computational domain and that of the accuracy of calculations themselves. Consequently, it is vital to update existing algorithms and create new ones for modern multicore architecture computers. However, improvement of existing programs on multiprocessing systems may bring some problems. It is essential to ensure that the results of a new approach provide the convergence of the same order as the results obtained in successive calculations. In order to construct a parallel algorithm without losing the calculation accuracy or stability, it is necessary to exploit physical laws written in conservative form.

Explosions of supernovae are a highly spectacular event in the Universe. Explanation of the core collapse in a supernova explosion mechanism is one of the most compelling and complicated problems of modern astrophysics. At an initial stage of the core collapse in an ongoing supernova research, the mechanisms of explosion has been connected to neutrino deposition and bounce shock propagation. Spherically, symmetrical numerical simulations have shown that the bounce shock appears at the distance of 10–30 km from the center; then, it moves at a radius of about 100–200 km and stalls without explosion. Farther investigations developed an extension of the same mechanism to 2D and 3D cases. Numerical simulations of 2D and 3D models have an additional feature connected to a development of neutrino driving convection deep inside after the shock. The complex calculations have shown with a sufficient level of confidence that this mechanism does not give a supernova explosion either.

A mechanism for core collapse supernova explosion, the *MR mechanism*, was suggested by Bisnovatyi-Kogan in 1970 [2], see also [3]. The main idea of the MR mechanism is to transform part of the rotational energy of presupernova into the radial kinetic energy (explosion energy). During a collapse the star rotates differentially. This differential rotation leads to a toroidal component of an appearing magnetic field and its amplification. The growth of the magnetic field means amplification of the magnetic pressure with time. A compression wave appears near a region of the extremum of the magnetic field. This compression wave moves outwards along a steeply decreasing density profile. In a short time it transforms into a fast MHD shock wave. When the shock reaches the surface of the collapsing star, it ejects part of the matter and energy increasing boundlessly. This ejection can be interpreted as an explosion of the core collapse supernova.

First 2D simulations of the rotating magnetized star collapse were presented in [4], with unrealistically large values of the magnetic field. The differential rotation and amplification of the magnetic field resulted in the formation of an axial jet. In 1D case, a star was represented as an infinite cylinder. A set of ideal

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MHD equations with self-gravitation in Lagrangian variables was used for these simulations. The initial magnetic field had only the r component. The differential rotation led to the appearance and amplification of the toroidal φ component of the magnetic field. Numerical simulations of 1D MR supernova had shown, that the toroidal field amplified due to the differential rotation that produced MHD shock wave which moved outwards. Part of the matter was ejected by the shock wave. The amount of the ejected energy $\approx 10^{51} erg$ is enough for the explanation of an supernova explosion. 1D simulations show that time of the evolution of a MR supernova t_{expl} depends on the relation of the initial magnetic E_{mag} and gravitational E_{grav} energies $\alpha = \frac{E_{mag}}{E_{grav}}$ as $t_{expl} \sim \frac{1}{\sqrt{\alpha}}$. It means that for real values of the magnetic field ($\alpha \approx 10^{-6-8}$) t_{expl} becomes rather large. Parameter α characterizes a stiffness of the MHD equations describing a MR supernova. The smallness of the parameter α is one of the main difficulties for the numerical simulation of a MR supernova. From the physical point of view, small α means the existence of two significantly different time scales – a very small acoustic time scale and a huge time scale proportional to the time of the magnetic field amplification.

More realistic model of magnetorotational supernova was calculated in 2D approximation. The star was represented by a rotating self-gravitating gaseous body. Results of the 2D simulations of the magnetorotational supernova are qualitatively different from 1D results. In the 2D case the magnetorotational instability (MRI) appears, leading to an exponential growth of all components of the magnetic field. MRI significantly reduces the time for the magnetorotational explosion. A toy model for the explanation of the MRI development in the magnetorotational supernova was suggested in the paper [5].

3D models of the magnetorotational supernova are more realistic and have no constraints related to the symmetry assumptions. 3D models allow us to simulate the magnetorotational supernova explosion in the case when rotational axis and the dipole magnetic field axis (if the dipole is taken as initial magnetic field) do not coincide (inclined rotator). The application of numerical method in Lagrangian variables, similar to the method used for the 2D case, leads to serious difficulties in 3D case [6].

In the 2D case, the matter of the star is slipping in φ direction. To produce the magnetorotational explosion the protoneutron star has to make thousands of revolutions. The rotation of the matter in the outer layers of the protoneutron star is highly differential. If the 3D Lagrangian grid consisting of tetrahedrons would be applied for the simulations, then in the region of a strong differential rotation the grid would require reconstruction at almost every time step. The reconstruction of the grid leads to the interpolation of the grid functions to a new grid structure. Frequent applications of the grid reconstruction procedure and interpolation of grid functions for the same parts of the Lagrangian grid can lead to a significant perturbation of the solution to initial set of MHD equations with self-gravitation.

2. Computational fluid dynamics

Computational fluid dynamics is a powerful approach in simulation of the complex gas flow occurring in astrophysical hydrodynamics. TVD, ENO, WENO, PPM schemes are referred to kinds of schemes that meet all these stated conditions and possess high resolution in regions of small perturbations combined with monotonicity in the domains of steep gradients. In this paper, we will consider TVD schemes of second order accuracy. The schemes of first order accuracy maintain monotony behavior, but often lead to strong smearing of shock wave fronts. Second order accurate of nonlinear schemes with the diminishing of total variation allow one to carry out calculations of high resolution and to prevent non-physical oscillations beyond shock wave fronts. The schemes of this type are of different order of accuracy in the domains with steep and low gradients [8,11]. Application of the these schemes in IIId case produces especially good results while simulating collapsing stars.

Equations that govern hydrodynamic motion are conservation laws for mass, momentum, [7] and energy. The conservation form of hydrodynamic equations in terms of Eulerian coordinate system is the following:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x_i}(\rho v_i) = 0, \tag{1}$$

$$\frac{\partial \rho v_i}{\partial t} + \frac{\partial}{\partial x_i}(\rho v_i v_j + P\delta_{ij}) = 0, \tag{2}$$

$$\frac{\partial e}{\partial t} + \frac{\partial}{\partial x_i}[(e + P)v_i] = 0. \tag{3}$$

The effect of gravitational field is omitted in Eqs. (1)–(3) as well as that of other sources of energy, for example, neutrino radiation. The equation of state can be written as following:

$$P = (\gamma - 1)\varepsilon, \tag{4}$$

here ρ is density, v is the vector of speed and P is pressure; moreover, the total energy is $e = \frac{1}{2}\rho v^2 + \varepsilon$.

A TVD scheme is applied to the Eqs. (1)–(3) [9,10]. A common restriction of oscillations is a nonlinear condition of stability. The discrete solution for TVD scheme can be defined as follows:

$$TV(u^t) = \sum_{i=1}^{N} |u^t_{i+1} - u^t_i|, \tag{5}$$

that is as the measure of total amount of oscillations.

Thus using second order accuracy for fluxes $F^{(2)t}_{i+1/2}$ across the cells boundaries, a nonlinear TVD scheme can be presented in another way. Second order fluxes are derived from first order accuracy fluxes $F^{(1)t}_{i+1/2}$ for the upwind scheme that apples a second order accuracy correction. A first order accuracy flux is obtained in turn from the flux mean values. The second order accuracy correction is introduced in order to bound spurious oscillations. Hence the number of oscillations on the current time step must not exceed the number of oscillations on the previous one, i.e. $TV(u_{i+1}) \leqslant TV(u_i)$.

Different flux limiters are used in order to limit oscillations, specifically, **minmod**, **superbee**, **vanLeer**. The former limiter chooses the smallest absolute value between the left and right corrections:

$$minmod(a, b) = \frac{1}{2}[sign(a) + sign(b)]\ min(|a|, |b|). \tag{6}$$

The **superbee** limiter chooses between the correction and 2 times the smallest correction, whichever is smaller in magnitude

$$superbee(a, b) = \begin{cases} minmod(a, 2b), & if\ |a| \geqslant |b|, \\ minmod(2a, b), & if\ |a| < |b|. \end{cases} \tag{7}$$

The **vanLeer** limiter is the most moderate of all limiters and allows one to find the harmonic mean between left and right corrections

$$vanleer(a, b) = \frac{2ab}{a + b}.$$

The test, proposed in [9], was used for checking the obtained computer program

$$u_0 = \begin{cases} -x \sin\left(\frac{3}{2}\pi x^2\right), & -1 \leqslant x < -\frac{1}{3}, \\ |\sin(2\pi x)|, & |x| < \frac{1}{3}, \\ 2x - 1 - \frac{1}{6}\sin(3\pi x), & \frac{1}{3} < x < 1. \end{cases} \tag{8}$$

Solution obtained by a TVD scheme with the **vanLeer** limiter (stared line) is presented in Fig. 1 The analytical solution is included for

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