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Optimal distribution of porous media to reduce trailing edge noise

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ABSTRACT

In a variety of technical applications the reduction of trailing edge noise is a matter of particular concern. The focus of the present study is to minimize trailing edge noise by adding an optimized porous medium to the existing geometry of the trailing edge. The optimization with respect to the noise reduction is based on iterative adjoint methods. Thus, the gradient of the objective function with respect to the control – the spacial distribution of the permeability – is obtained by solving an adjoint equation backwards in time. The mathematical framework of the governing compressible and porous Navier–Stokes equations and its corresponding adjoint equations, to obtain the gradient, is presented and validated on a two-dimensional example. The iterative method has proven effective an efficient in minimizing aeroacoustic noise within only five iterations. This technique can be used in aeroacoustic and other applications where the (shape) optimization of (porous) material is desirable.

1. Introduction

In many technical applications the reduction of trailing edge noise, is a matter of particular concern. Trailing edge noise can be found in any technical application where fluid flows past a rigid trailing edge. Especially sharp edges contribute to loud noise sources with a broadband spectrum [11]. These conditions can be found e.g. at the high lift configuration of aircraft during landing or takeoff at moderate subsonic Mach numbers. Rather blunt edges, which can be found e.g. at the aircrafts landing gear, contribute to a narrow-band frequency noise due to the dominated vortex-shedding mechanism [2,19].

The focus of the present research project lies in the minimization of trailing edge noise by a modification of the trailing edge. In contrast of changing the edge geometry which has been performed by a series of researchers, like [22,20], we will follow a different approach by adding a porous medium at the trailing edge to suppress vortex induced noise. The idea is to control the wake of the trailing edge with the porous medium such that its emanated noise will be minimized in a pre-defined region. In Fig. 1, a sketch of the computational domain with the implemented boundary conditions and the position of the trailing is visualized. The geometry of the trailing edge is marked as a black plate with a hight h and a length $8h$. At the end of the plate, a shaded area of the same hight as the plate and a total length of $4h$ marks the area where the optimization algorithm can place the porous medium. The emanated noise of the trailing edge including the porous medium is measured along the dashed double line in a distance of $14h$ from the center of the solid plate.

Experimental results [10] on trailing edge noise, showed a possible noise reduction of up to 10 dB for the application of a specific porous medium. The results show, that the choice of the porous medium is of particular importance. Not all porous media investigated showed an overall noise reduction. Some porous media even amplified frequencies compared to a rigid reference body. It could be also shown that there is a vast difference in amount of the noise reduction. The differences lie in the range of 0 dB and the maximum of 10 dB. In a numerical study [15], on the application of porous media to cavity noise, a reduction of up to 10 dB could be reached, too. Again, the choice of the porous medium and its spatial distribution seemed to be very crucial. An inauspicious choice of the porous medium could again increase the noise level.

In the literature, porous media are not only used for aeroacoustic applications, but can be found in any flow control application. For instance, Bruneau and Mortazavi [3] controlled the vortex shedding of the flow past a cylinder with a porous coating. In another numerical study [23], the Mack mode was stabilized in a Mach 6 boundary layer flow by means of a porous surface.

The porous medium investigated in the present study, is modeled by a volume force, similar to Darcy's law for incompressible flow. For the present aeroacoustic application the volume force is implemented in the full compressible Navier–Stokes equations leading to a new set of porous flow equations.

The porous medium is characterized by only two parameters, the porosity ϕ and the permeability $\boldsymbol{K}$. Whereas ϕ describes the

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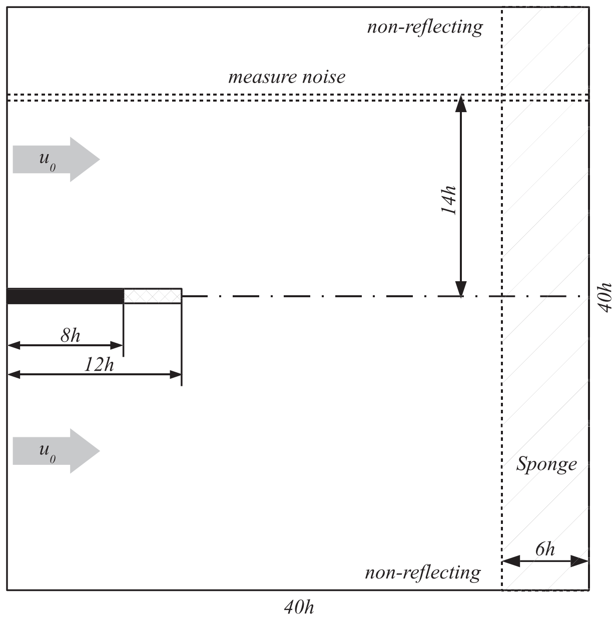

Fig. 1. Sketch of the two-dimensional computational domain with trailing edge and implemented boundary conditions. The hight of the trailing edge is h. Solid part of the trailing edge is filled in black, cross-shaded part is porous and can be optimized.

volume ratio of void space V_f to the volume of the whole porous material ($\phi = V_f/V$). Thus, a porosity equal to one represents void space only, and a porosity of zero a solid material, where no fluid could penetrate. The latter extreme would causes a singularity in the porous flow equations, thus only values of $0 < \phi \leqslant 1$ are feasible.

The second parameter $\boldsymbol{K}$ stands for the permeability of the material and is a symmetric and positive definite tensor

$$\boldsymbol{K} = \begin{pmatrix} \kappa_{11} & \kappa_{12} & \kappa_{13} \\ \kappa_{21} & \kappa_{22} & \kappa_{23} \\ \kappa_{31} & \kappa_{32} & \kappa_{33} \end{pmatrix}.$$

The entries in that tensor can reach values of zero for a solid material which is not permeable (solid) and infinity for a material with no influence on the fluid (zero drag, void material). To keep the freedom in the design process of the porous medium, the two coefficients ϕ and $\boldsymbol{K}$ are supposed to be functions of space and time. Anyway, since the realization of a time dependent structure of a porous medium for technical applications is hard to realize, only the space dependence will be considered ($\phi = \phi(\boldsymbol{x})$, $\boldsymbol{K} = \boldsymbol{K}(\boldsymbol{x})$).

Numerical [15] and experimental studies [10] show that it is by no means clear how to choose the porous medium (ϕ and $\boldsymbol{K}$) to reduce trailing edge noise. To this end, an optimization technique will be used to obtain the optimal distribution of the porosity and permeability. Due to the space-dependence of the design parameters, the number of parameters to control can be in the order of several thousand since ϕ and $\boldsymbol{K}$ have to be determined at every grid point in the control region. To handle this amount of degrees of freedom in the optimization algorithm, adjoint methods will be used in an iterative design process to obtain the gradient information of the objective function (noise). The adjoint equations are based on the full compressible porous Navier–Stokes equations and are derived in a continuous manner without any further simplifications. Since the control parameters are constraint ($0 < \phi \leqslant 1$ and $\boldsymbol{K} < 0$), due to their physical properties, a method based on *slack* variables will be used. Some recent reviews on adjoint based methods for aeroacoustic noise minimization and related formulations can be found in [5,8,29].

In the present investigation, the mathematical framework of an adjoint-based optimization algorithim to optimize a porous medium will be presented. The method will be applied to minimize trailing edge noise. The organization of the paper is as follows: in the Section 2, we will give a general description of the porous flow equations to implement a porous material in an existing flow solver. This is followed by deriving the governing equations of the iterative optimization scheme with porous adjoint equations. In Section 3 we show some results of a trailing edge with an optimized porous medium. In Section 4 the main results of the study will be summarized.

2. Mathematical framework

To include a porous medium in the simulation, the equations of continuity, momentum and energy have to be modified. The main idea is based on a relation found by Darcy [7] to relate the flow velocities and the pressure gradient with the permeability of the porous medium in a linear way. This relation, called Darcy's law, has been validated in several experiments and reads in the present terminology:

$$\boldsymbol{v} = -\frac{\boldsymbol{K}}{\mu}\nabla\boldsymbol{p}, \tag{1}$$

with the so called Darcy velocity $\boldsymbol{v} = \phi\boldsymbol{u}$. This additional volume force is added to the momentum equation and acts like a source term damping all velocities in the porous medium. In addition to that, the density of the fluid in the porous medium, averaged in an infinitesimal small control volume, will change [9]. It is related to the porosity: $\phi = V_f/V = \rho/\rho_f$, with the density and volume of the fluid, ρ_f and V_f, respectively, and V as the control volume (cf. Fig. 2). This second fundamental law for porous media has to be included in the whole derivation of the compressible porous flow equations.

In the literature, different ways to include Darcy's law can be found. Especially the choice of the velocities ($\boldsymbol{v}$ or $\boldsymbol{u}$) and the placement of the porosity ϕ is not unique. Most applications deal with the incompressible Navier–Stokes equations where terms simplify [12,13,3]. In the compressible case [18,14], use a set of modified Navier–Stokes equations including Darcy's law. As they use a different formulation for the energy equation, without taking into account the transformation of kinetic energy into entropy, we will present the porous Navier–Stokes equations for compressible flow including an equation for the entropy in the following section.

2.1. Governing porous flow equations

Replacing consequently the density ρ by $\phi\rho_f$ and including Darcy's law (1), one obtains a set of porous flow equations for compressible fluids, based on the Navier–Stokes equations,

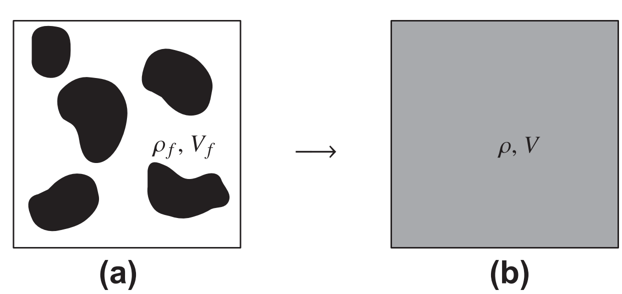

Fig. 2. Replacing the density of the fluid ρ_f in a real porous medium (a) by the density ρ in a homogenized volume V (b). The porosity is defined as $\phi = V_f/V = \rho_f/\rho$.