Computers & Fluids 71 (2013) 19–27

Contents lists available at SciVerse ScienceDirect

Computers & Fluids

journal homepage: www.elsevier.com/locate/compfluid

A remapping-free, efficient Riemann-solvers based, ALE method for multi-material fluids with general EOS

Guoxi Ni *, Song Jiang, Shuanghu Wang

LCP, Institute of Applied Physics and Computational Mathematics, P.O. Box 8009, Beijing 100088, China

ARTICLE INFO

Article history:
Received 28 August 2010
Received in revised form 26 September 2011
Accepted 2 October 2012
Available online 23 October 2012

Keywords:
General EOS
Inverse Hermite interpolation
Diffusive mesh velocity
Moving mesh
Multi-material fluids

ABSTRACT

Based on an efficient Riemann solver, a remapping-free ALE method (RALE) for multi-material fluids with general equations of state is proposed. The basic idea of constructing the RALE is to couple the Lagrangian method with a remapping-free ALE-type method. In order to keep the sharpness of a material interface, the Lagrangian formulation is employed for tracking the material interface, where the Lagrangian velocity of nodes and Lagrangian fluxes are designed. In single material regions, the numerical fluxes are constructed on moving meshes which move nodes to the regions with large gradients to increase the numerical accuracy, and the explicit remapping stage is avoided because of the new discrete scheme. The inverse Hermite interpolation argument is employed in solving the Riemann problem with general EOS, consequently, reducing iteration steps greatly and resulting in an efficient and robust Riemann solver. A number of numerical examples are presented.

1. Introduction

Traditionally, numerical methods for fluid dynamics have fallen into two camps: Eulerian methods and Lagrangian methods. The Eulerian method is robust, easy in coding, and capable of running under severe flow conditions, such as large flow deformation, and has been well developed and widely applied (see e.g. [1–3]), but may result in badly smeared material interfaces for multi-material flows or wave structure due to numerical diffusion. As to the Lagrangian method, the meshes move with the fluids and there is no mass flux across moving interfaces, material interfaces remain exact as they travel with cells, the Lagrangian method is therefore capable of producing sharp material interfaces, but may lead to mesh distortion and tangling, causing inaccuracy and even breakdown of computation. However, due to the distinguishable features of the Lagrangian method, such as the capability of capturing material interfaces and dealing with free boundaries, the method has been developed considerably and applied to a variety of scientific and engineering problems in the past decades [4–9].

In order to improve numerical accuracy, the development of mesh movement methods have recently attracted much attention. To avoid mesh distortion and tangling in a Lagrangian method, a well-known Arbitrary Lagrangian–Eulerian (ALE) technique has been developed, see for example, [10–17]. In the ALE method, the numerical algorithm covers from Lagrangian to Eulerian methods through a rezoning step to fix a distorted mesh. Due to the smoothing techniques in the rezoning step, a regular mesh can be obtained, but numerical errors could be easily induced by the remapping which transfers flow variables from a old mesh to a new one. A similar method is the so-called moving mesh method studied intensively in the recent years. Generally, the moving mesh method consists of two steps: after each Eulerian step, a new mesh is generated according to certain requirements, such that nodes move to the regions with large changes of the physical quantities to increase accuracy of numerical results. In [18,19], the authors move nodes according to a flow gradient-dependent monitor function, so that the nodes of a new mesh can concentrate in the regions with large gradients. Then, with the introduction of the new mesh, all flow variables are interpolated from the old mesh to the new one. In [20] the authors proposed another moving mesh technique by defining adaptive grid speed in each time step to improve the resolution of shocks and contact discontinuities. The key point in [20] is to construct the finite volume scheme on time–space polyhedron or quadrilateral meshes and then numerical fluxes on interfaces. Recently, this method was further developed by many researchers, see for example [21,22]. In [23], the authors developed an unified and efficient moving mesh method, which is one of ALE type schemes, but remapping-free (without explicit remapping), the basic idea is to only construct the numerical fluxes across cell interfaces of a moving mesh and then to update the variables in the inertial frame.

For the multi-material flow simulations, the development of accurate numerical schemes has become one of the most important research topics in computational fluid dynamics, and a number of

* Corresponding author. Tel.: +86 10 62014411 2968.
E-mail addresses: gxni@iapcm.ac.cn (G. Ni), jiang@iapcm.ac.cn (S. Jiang), wang_shuanghu@iapcm.ac.cn (S. Wang).

0045-7930/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compfluid.2012.10.005

schemes have been proposed. One of the typical methods is to solve an extended conservative system, where additional conservation equations are introduced to the original fluid equations to capture a fluid interface, such as the level set functions, the mass fractions and the ratio of specific heats (γ-model), see, for example, [24–26]. In order to maintain the pressure equilibrium and the mass fraction positivity, and to eliminate spurious oscillations near material interfaces, several non-conservative approaches to capture the contact discontinuities using an additional non-conservative governing equation have been proposed [27,28]. In [29], the ghost fluid methods are presented, where the ghost cells and isobaric fix techniques are used to keep the density profile from smearing. Unfortunately, it cannot work consistently and efficiently when applied to a strong shock. To overcome this difficulty, a modified ghost fluid method is proposed and developed for more robustness and consistency [30]. By incorporating the conservative γ-model into the finite volume BGK scheme, a γ-model BGK method for compressible multicomponent flow computation was proposed in [31,32], and an efficient and accurate BKG scheme for multifluids is obtained.

In this paper, we will propose a remapping-free ALE method for multi-material fluids (abbrev. RALE) with general equations of states by employing an efficient Riemann solver. The basic idea of constructing the RALE is to couple the Lagrangian method of tracking material interfaces with the remapping-free ALE-type method within a single material domain. In order to keep the sharpness of material interfaces, the Lagrangian velocity and fluxes are constructed on the material interfaces, using the node-centered solver developed by Maire et al. [6], and the velocity of the mesh cells away from the material interfaces are determined by diffusive mechanism (velocity), for which many approaches can be used to get the diffusive velocity [33,34]. Then, the fluxes are constructed on the moving meshes, so that both accuracy and robustness are taken into account. Consequently, a generalized ALE method for multi-material flows is constructed.

An important feature of the RALE is the use of an efficient Riemann solver for general EOS to get the velocity of mesh movement and fluxes. For perfect gases, the Newton iteration can be used to get an exact Riemann solution, but for a non-perfect gas, it could not work well in many cases. To circumvent this drawback, here we employ an inverse Hermite interpolation argument [35] for general EOS. In the inverse Hermite interpolation argument, we first approximate the inverse function of the pressure by parabola, and the root of the inverse function can be easily found by solving algebra equations. Then, we obtain an exact solution by iteration over smaller domains. In this manner, the number of iterations is greatly reduced, thus the RALE is more efficient and robust than the linear interpolation.

This paper is organized as follows. In Section 2, the Riemann problem for general EOS is studied. In Section 3, we construct the RALE method for multi-material fluids, and the method of determining the mesh velocity is presented, while in Section 4, numerical examples to demonstrate the accuracy and robustness of the proposed scheme are given.

2. Riemann solver for general equations of state

To study the Riemann solver for general equations of state, we begin with the one-dimensional system. The motion of one-dimensional compressible inviscid flows is described by the conservation laws of mass, momentum and energy, which can be written in the following differential form:

$$\begin{cases} \frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0, \\ \frac{\partial \rho u}{\partial t} + \frac{\partial (\rho u^2 + p)}{\partial x} = 0, \\ \frac{\partial \rho E}{\partial t} + \frac{\partial ((\rho E + p)u)}{\partial x} = 0, \end{cases} \tag{2.1}$$

where $\rho E = \rho e + \frac{1}{2}\rho u^2$, and equation of state $p = p(\rho, e)$ is needed to close the system.

In the construction of numerical fluxes of our scheme, we need to solve the Riemann problem for the system (2.1), and we will present an efficient algorithm for the Riemann problem of (2.1) with general EOS, which will be used in the next section.

2.1. General equations of state

In general, an equation of state (EOS) relates three thermodynamic variables: the pressure, the density and the internal energy. The equation of state varies with materials, and it is a complex function of the molecular and atomic structure of a given substance, the most convenient form of the equation of state for a perfect gas is $p = (\gamma - 1)\rho e$, the pressure is a simple function of the density and the internal energy, where γ is the ratio of the specific heat at constant pressure to that at constant volume, this form is commonly used in theoretic analysis and numerical methods.

We will consider the following three kinds of EOS. The first one is for a stiffened gas, which can be written in the following form:

$$p = (\gamma - 1)\rho e - \gamma p_\infty, \tag{2.2}$$

where p_∞ is the prescribed pressure-like constant and can be used to describe the material property, for example, $\gamma = 5.5$, $p_\infty = 4921.15$ for water.

The second one is the binomial type EOS of the form:

$$p = (\gamma - 1)\rho e + c_0^2(\rho - \rho_0), \tag{2.3}$$

where ρ_0, c_0 are the substance parameters. It is a much more realistic equation of state for solids.

The third one is the so-called Mie-Gruneisen type EOS in the following form:

$$p = (\gamma - 1)\rho e + \frac{\rho_0 c_0^2}{\mu - 1}\left\{(\gamma - 1)(\delta - 1) + \frac{\gamma - \mu}{\mu}(1 - \delta^\mu)\right\}, \tag{2.4}$$

where $\delta = \rho/\rho_0$. Eq. (2.4) is designed to duplicate the linear shock particle velocity relation at low pressure and to extrapolate the Thomas–Fermi limit at high pressure. There are also parameters that allow (2.4) to describe the unloading of a shocked material to vapor phase.

An approximate form of (2.4) by the binomial EOS is given in [35], where the parameters $\tilde{\gamma}, \tilde{c_0}, \widetilde{\rho_0}$ in (2.3) are given by:

$$\tilde{\gamma} = 1 + \frac{1}{\rho}\frac{\partial p}{\partial e}, \quad \widetilde{c_0}^2 = \frac{\partial p}{\partial \rho} - \frac{e}{\rho}\frac{\partial p}{\partial e}, \quad \widetilde{\rho_0}\widetilde{c_0}^2 = \rho\frac{\partial p}{\partial \rho} - p,$$

this approximation works satisfactorily in many cases.

2.2. Inverse Hermite interpolation for the Riemann problem

In out RALE scheme, a Riemann solver is needed in two aspects: on one hand, for the finite volume scheme in a single material region, we need to solve the Riemann problem to get the numerical fluxes on the moving meshes. On the other hand, for a material interface, we also have to solve the Riemann problem across the material interface to get the nodal velocity and the Lagrangian fluxes as well. For a perfect gas, the Newton iteration is sufficient to get the Riemann solution, but for a non-perfect gas, the Newton iteration will diverge in many cases, in particular, for the complex EOS. In this subsection, we will present an efficient inverse Hermite interpolation to solve the Riemann problem with the above EOS.

Suppose that we have two constant states, namely the left is (ρ_1, u_1, p_1), and the right is (ρ_2, u_2, p_2), the basic configurations of the Riemann problem include two rarefaction waves, two shock