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Heat and mass transfer of MHD second order slip flow

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ABSTRACT

The present paper is devoted to the analysis of MHD flow and heat transfer over permeable stretching/shrinking surfaces taking into account a second order slip model. The purpose is to extract exact analytical solutions for the flow and heat valid under various physical conditions. Particular attention is paid for the effects of magnetic field on the second order slip flow conditions. Results of the present analysis in the absence of magnetic field are in excellent agreement with those available in the literature. The velocity and temperature profiles, skin friction coefficient and Nusselt number are easily examined and discussed via the closed forms obtained. For all the considered parameters, unique solutions are detected for the flow over a stretching sheet, whereas solutions turn out to be multiple for some combinations of parameters in the case of flow over a shrinking sheet. Compared to the no-slip case, as the slip is increased, the region of multiple solutions is found to extend.

1. Introduction

The fluid flow over a continuously stretching or shrinking surface finds many important applications in engineering processes, such as polymer extrusion, drawing of plastic films and wires, glass fiber and paper production, manufacture of foods, crystal growing, liquid films in condensation process, etc., see Fisher [1]. The studies of Sakiadis [2] and Crane [3] are the pioneering ones in this area. A dozen of flow properties were later investigated by the followers [4–10] using no-slip condition on the wall. However, as stated in [11], when the fluid is particulate such as emulsions, suspensions, foams and polymer solutions, the no-slip condition is inadequate. In such cases the suitable boundary condition is the partial slip. Wang [12,13] discussed the partial slip effects on the planar stretching flow. Fang et al. [14–17] found analytical solutions representing the hydromagnetic flow.

The applied magnetic field may play an important role in controlling momentum and heat transfers in the boundary layer flow of different fluids over a stretching/shrinking sheets. Including the magnetic field, a variety of physical properties for the flow and heat transfer over stretching/shrinking sheet were analytically investigated in Turkyilmazoglu [18–20], see also the references therein.

Recently, Fang et al. [21] considered the effects of second order slip on the flow of a shrinking sheet and the case of stretching sheet was studied by Vajravelu et al. [22]. It is seen that in the literature a proper analysis of second order slip flow and heat transfer over a stretching/shrinking sheet with magnetic field has not been investigated yet. Therefore, the purpose of this attempt is to investigate the influence of second order slip on the behavior of flow and thermal transport of an electrically conducting fluid over a permeable stretching/shrinking sheet with two general heating processes namely the PST and the PHF cases [18,20]. We hence derive closed-form analytical solutions for the velocity and temperature profiles as well as the skin friction and heat transfer coefficients of physical importance. We should remark that although the analysis is similar, the present study is totally different from those of [18,20] in that second order slip flow together with extra form of analytical solutions are presented here. Moreover, the present work is different from that of [21] in that the MHD effects and the resolution of energy field are not accounted for in the shrinking sheet study of [21]. In addition to this, the recent investigation of [22] over a stretching sheet was extended here to cover for the magnetic field effects for not only stretching but also shrinking surfaces. Another important point is that the energy equation was also solved analytically here, which was treated numerically in [22].

The arrangement of the paper is in the following. The problem is formulated in Section 2. The analytical solutions for both flow and temperature fields are presented in Section 3. Section 4 contains results and discussions. The concluding remarks are eventually given in Section 5.

2. Formulation of the problem

Let us consider a steady, two-dimensional laminar slip flow over a continuously stretching or shrinking sheet in an electrically

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Nomenclature

(A,B)	dimensional first and second order velocity slips	T_w	wall temperature
(b_1,b_2)	constants	T_∞	uniform temperature
B_0	uniform magnetic field strength	u	velocity component in x-direction
c	a positive constant	v	velocity component in y-direction
c_p	specific heat at constant pressure	v_w	dimensional suction or injection parameter
d	stretching/shrinking parameter	(x,y)	longitudinal and transverse directions
f	dimensionless self-similar velocity		
k	thermal conductivity	*Greek symbols*	
L	incomplete Laguerre polynomial	η	a boundary layer coordinate
M	magnetic interaction strength parameter	θ	a scaled temperature
Nu	Nusselt number	(γ,δ)	first and second order velocity slip parameters
p	a variable	ρ	density of the fluid
Pr	Prandtl number	ν	kinematic viscosity of the fluid
Pr^*	a scaled Prandtl number	λ	exponential constant
s	dimensionless suction or injection parameter	λ_i	dummy variables
S	a variable	Λ	a variable
t	a variable	σ	electrical conductivity of the fluid
T	temperature		

Table 1
Comparison of the values of λ for $d = -1$, $M = 0$ and $\delta = -1$ for several s and γ with those of [21] in parenthesis. Upper and lower branch solutions are presented.

s	$\gamma = 0$	$\gamma = 1$	$\gamma = 3$	$\gamma = 10$
Upper				
2	1.8832035(1.8832)	1.9212896(1.9213)	1.9519690(1.9520)	1.9795599(1.9796)
3	2.9655726(2.9656)	2.9737635(2.9738)	2.9822017(2.9822)	2.9916152(2.9916)
5	4.9922728(4.9923)	4.9935252(4.9935)	4.9951096(4.9951)	4.9973652(4.9974)
10	9.9990096(9.9990)	9.9990989(9.9991)	9.9992365(9.9992)	9.9995024(9.9995)
Lower				
2	0.53101006(0.5310)	0.40052899(0.4005)	0.29676823(0.2968)	0.18882968(0.1888)
3	0.33720300(0.3372)	0.27228263(0.2723)	0.21301670(0.2130)	0.14289554(0.1429)
5	0.20030956(0.2003)	0.17232557(0.1723)	0.14226228(0.1423)	0.10102758(0.1010)
10	0.10000990(0.1000)	0.09173797(0.0917)	0.08073383(0.0807)	0.06197570(0.0620)

conducting quiescent fluid coinciding with the plane $y = 0$, the flow being confined to $y > 0$. A uniform external magnetic field of strength B_0 is supposed to act in the direction perpendicular to the sheet. Following the analysis in [20], the governing dimensional flow and energy equations of this problem read (see also [21,22])

$$\begin{aligned} &u_x + v_y = 0,\\ &uu_x + vu_y = \nu u_{yy} - \frac{\sigma B_0^2}{\rho}u,\\ &uT_x + vT_y = \frac{k}{\rho c_p}T_{yy}. \end{aligned} \tag{2.1}$$

Eq. (2.1) are considered with the following boundary conditions

$$\begin{aligned} &u(x,0) = dcx + Au_y(x,0) + Bu_{yy}(x,0), \quad v(x,0) = v_w, \quad u(x,\infty) = 0,\\ &T(x,0) = T_w = T_\infty + b_1x^2(\text{PST case}), \quad -kT_y(x,0) = b_2x^2(\text{PHF case}),\\ &T(x,\infty) = T_\infty, \end{aligned} \tag{2.2}$$

Table 2
Comparison of the values of $-\theta'(0)$ and $\theta(0)$ for $d = 1$ and $M = 0$ for various parameters with those of [22] in parenthesis.

Pr	γ	δ	s	$-\theta'(0)$	$\theta(0)$
1	1	−1	2	2.09847360(2.098476)	0.47653685(0.476536)
1	1	−2	2	2.06423598(2.064238)	0.48444074(0.484440)
1	1	−3	2	2.04775583(2.035026)	0.48833947(0.488339)
1	3	−1	2	2.06471031(2.064712)	0.48432945(0.484329)
2	1	−1	2	4.13198978(4.131991)	0.24201415(0.242014)
3	1	−1	2	6.14919431(6.149186)	0.16262293(0.162623)

where $d = 1$ denotes stretching and $d = -1$ denotes shrinking sheets, respectively. From the first of (2.2), it is worth to mention that the model taken into account is governed by a second order slip, that is, $B \neq 0$. It is also noticed that two types of general heating processes, namely, the prescribed surface temperature (PST) and the prescribed wall heat flux (PHF) are considered.

3. Exact analytical solutions

3.1. Solution of the flow field

Similar to [20], we introduce the subsequent similarity transformations

$$\eta = y\sqrt{\frac{c}{\nu}}, \quad u = cxf'(\eta), \quad v = -\sqrt{c\nu}f(\eta), \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \tag{3.3}$$

which gives $v_w = -\sqrt{c\nu}f(0)$, and the governing equations of motion (2.1) and (2.2) is reduced to the similarity form

$$\begin{aligned} &f''' + ff'' - f'^2 - Mf' = 0,\\ &\theta'' + Pr(f\theta' - 2f'\theta) = 0, \end{aligned} \tag{3.4}$$

together with the boundary conditions

$$\begin{aligned} &f(0) = s, \quad f'(0) = d + \gamma f''(0) + \delta f'''(0), \quad f'(\infty) = 0,\\ &\theta(0) = 1(\text{PST case}), \quad \theta'(0) = -1 \ (\text{PHF case}), \quad \theta(\infty) = 0. \end{aligned} \tag{3.5}$$

Here $\gamma = A\sqrt{\frac{c}{\nu}}$ is the first order velocity slip parameter and $\delta = B\frac{c}{\nu}$ is the second order velocity slip parameter. It should be noticed that when $M = 0$ and $d = 1$, Eqs. (3.4) and (3.5) reduce to those recently