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New exact solutions for the (2 + 1)-dimensional Sawada–Kotera equation

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ABSTRACT

A new test function is proposed to construct new exact solitary solution for nonlinear evolution equation. The (2 + 1)-dimensional Sawada–Kotera equation is employed as an example to illustrate the effectiveness of the suggested method and some new wave solutions with three different velocities and frequencies are obtained. Obviously, the method can be applied to solve other type of nonlinear evolution equations as well.

1. Introduction

Many phenomena in physics and in the nonlinear science can be modeled by a class of integrable nonlinear evolution equations. Consequently, construction of traveling wave solutions of nonlinear equations plays an important role in the study of nonlinear phenomena. Nowadays, with the rapid development of software technology, solving nonlinear evolution equations via symbolic computation is taking an increasing role due to its efficiency, accuracy and its easy use. Over the last few decades, directly searching for exact solutions of nonlinear partial differential equations, some new powerful solving methods have been developed, such as $\left(\frac{G'}{G}\right)$ - expansion method [1], exp-function method [2,3], Hirota bilinear method [4–6], F-expansion method [7–9], tanh method [10,11], homoclinic test method [12,13], Homotopy perturbation method [14,15], and so on. Recently, Dai and Wang has proposed a novel approach to study the result when three waves of different frequencies and different propagation velocity meet interaction, namely, extended three-soliton method [16–18], they study the behavior of dynamics of higher dimensional nonlinear integrable system by using extended three-soliton method and obtained some new type solutions of integrable nonlinear evolution equations.

In this work, the multi-wave method is proposed to seek for new exact solitary solution for nonlinear evolution equation. The Sawada–Kotera equation is employed as an example to illustrate the effectiveness of the suggested method and some new wave solutions with four different velocities and frequencies are obtained, include periodic solitary wave solutions, bright soliton wave solutions, W-type wave solutions, etc.

The (2 + 1)-dimensional Sawada–Kotera equation was proposed by Konopelcheno and Dubrovsky in [19]:

2. The bilinear form of the (2 + 1)-dimensional Sawada–Kotera equation

$$u_t = \left(u_{xxxx} + 5uu_{xx} + \frac{5}{3}u^3 + 5u_{xy}\right)_x - 5\int u_{yy}dx + 5uu_y + 5u_x\int u_y dx. \quad (1)$$

It was widely used in many branches of physics, such as conformal field theory, two-dimensional quantum gravity gauge field, theory and nonlinear science Liuvill flow conservation equations [20]. The symmetry analysis and finding the exact solutions of Eq. (1) were studied in Refs. [21–24]. In this work, we are going to seek for more new type solutions of Eq. (1) by using the multi-wave method.

Let's suppose

$$u = -2(\ln F)_{xx}, \quad (2)$$

where $F = F(x, y, t)$ is an unknown real function.

Substituting Eq. (2) into Eq. (1) we can reduce Eq. (1) into the bilinear form as follows:

$$(D_x^6 + 5D_yD_x^3 - 5D_y^2 + D_xD_t)(F \cdot F) = 0, \quad (3)$$

where the D-operator is defined by

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$$D_x^m D_y^n(a\cdot b)=\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)^m\left(\frac{\partial}{\partial y}-\frac{\partial}{\partial y'}\right)^n a(x,y)\cdot b(x',y')\Big|_{y=y',\ x=x'}. \tag{4}$$

Eq. (3) can be also rewritten as

$$F_{xxxxxx}F-6F_{xxxxx}F_x+15F_{xxxx}F_{xx}-10F_{xxx}^2+5F_{xxxy}F-5F_{xxx}F_y$$
$$-15F_{xxy}F_x+15F_{xx}F_{xy}-5F_{yy}F+5F_y^2+F_{xt}F-F_xF_t=0. \tag{5}$$

3. The new exact solutions of the (2+1)-dimensional Sawada–Kotera equation

In this case we propose a novel test function of extended three-soliton method:

$$F(x,y,t)=a_1\cos\xi_1+a_2\cosh\xi_2+\exp(-\xi_3)+a_3\exp(\xi_3). \tag{6}$$

where $\xi_i=k_ix+h_iy+w_it,\ k_i,h_i$ is the real parameters of amplitude of the ith soliton. w_i is the wave speed, $a_i (i=1,2,3)$ are arbitrary constants.

Substituting Eq. (6) into Eq. (5), and equating the coefficients of all powers of $\sin\xi_1\exp(\pm\xi_3)$, $\sin\xi_1\sinh\xi_2$, $\cos\xi_1\exp(\pm\xi_3)$, $\cos\xi_1\cosh\xi_2$, $\sin\xi_2\exp(\pm\xi_3)$, $\cos\xi_2\exp(\pm\xi_3)$, $\exp(0)$, we can obtain a set of algebraic equations for $a_1, a_2, a_3, k_i, h_i, w_i$ $(i=1,2,3)$,

$$a_1a_3(-10h_1h_3-20k_1^3k_3^3+6k_3^5k_1-15h_1k_1^2k_3+k_3w_1+6k_1^5k_3$$
$$+k_1w_3+5k_3^3h_1-5k_1^3h_3+15k_3^2h_3k_1)=0,$$

$$a_1(15h_1k_1^2k_3-15k_3^2h_3k_1-5k_3^3a_1h_1+5k_1^3h_3+10h_1h_3$$
$$+20k_1^3k_3^3-6k_1^5k_3-k_3w_1-6k_3^5k_1-k_1w_3)=0,$$

$$a_1a_2(5k_2^3h_1-15h_1k_1^2k_2+6k_1^5k_2+k_2w_1+k_1w_2-20k_1^3k_2^3$$
$$-10h_1h_2-5k_1^3h_2+15h_2k_2^2k_1+6k_2^5k_1)=0,$$

$$a_1a_3(-15k_1^2k_3h_3+5h_1k_1^3-k_1^6+5h_1^2-15k_3^4k_1^2+k_3w_3+5k_3^3h_3$$
$$-15k_3^2h_1k_1-w_1k_1+15k_1^4k_3^2+k_3^6-5h_3^2)=0,$$

$$a_1(k_3^6-w_1k_1+5k_3^3h_3+15k_1^4k_3^2-15k_3^4k_1^2+5h_1k_1^3+k_3w_3$$
$$-a_1k_1^6+5h_1^2-5h_3^2-15k_1^2k_3h_3-15k_3^2h_1k_1)=0,$$

$$a_1a_2(w_2k_2-15k_2^2h_1k_1-w_1k_1-k_1^6+5h_1^2+k_2^6-5h_2^2+5h_2k_2^3$$
$$+15k_1^4k_2^2-15k_1^2h_2k_2+5h_1k_1^3-15k_2^4k_1^2)=0,$$

$$a_2a_3(-5h_3^2+k_3^6+15k_2^2k_3h_3+5h_2k_2^3+5k_3^3h_3+15k_3^2h_2k_2$$
$$+15k_2^4k_3^2+15k_3^4k_2^2-5h_2^2+k_2^6+w_2k_2+k_3w_3)=0,$$

$$a_2(15k_2^2k_3h_3-5h_3^2+15k_3^2h_2k_2+k_3^6-5h_2^2+k_2^6+w_2k_2$$
$$+15k_3^4k_2^2+5h_2k_2^3+15k_2^4k_3^2+5k_3^3h_3+k_3w_3)=0,$$

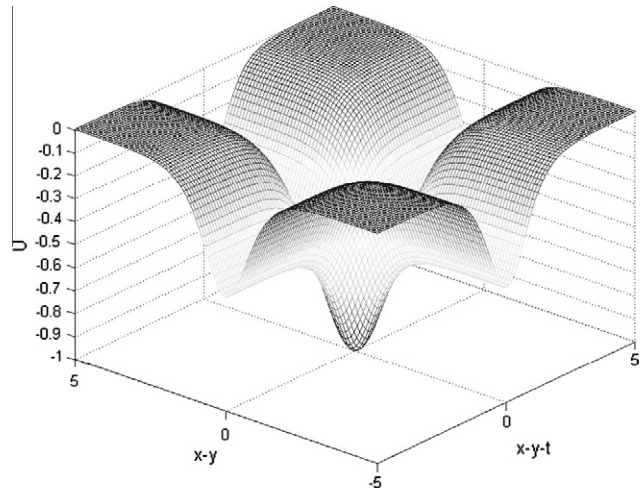

Fig. 1. Double solitons. (The figure of u_1 at $a_3=2$, $k_3=0.5$, $h_3=2$, $t=0$.)

$$a_2a_3(-15k_3^2h_3k_2+10h_2h_3-15h_2k_2^2k_3-6k_3^5k_2-5k_2^3h_3$$
$$-6k_2^5k_3-k_2w_3-20k_2^3k_3^3-k_3w_2-5k_3^3h_2)=0,$$

$$a_2(20k_2^3k_3^3-10h_2h_3+15h_2k_2^2k_3+15k_3^2h_3k_2+5k_3^3h_2+6k_3^5k_2$$
$$+k_2w_3+6k_2^5k_3+k_3w_2+5k_2^3h_3)=0,$$

$$a_3(-20h_3^2+64k_3^6+80k_3^3h_3+4k_3w_3)+a_1^2(5h_1^2-16k_1^6+20k_1^6h_1-k_1w_1)$$
$$+a_2^2(20h_2k_2^3-5h_2^2+16k_2^6+w_2k_2)=0.$$

Solving this system of algebraic with the aid of Maple, we obtain nine sets of solutions as follows,

3.1. Case 1

$$a_1=0,\ a_2=0,\ w_3=-\frac{-5h_3^2+16k_3^6+20k_3^3h_3}{k_3}. \tag{7}$$

where a_3, k_3, h_3 are free parameters. Substituting (7) into Eq. (6) with Eq. (2), we can obtain the soliton solution of Eq. (1) as

$$u_1=-2k_3^2+2k_3^2[\tanh(\xi_3+\ln\sqrt{a_3})]^2, \tag{8}$$

here $\xi_3=k_3x+h_3y-\frac{-5h_3^2+16k_3^6+20k_3^3h_3}{k_3}t$, a_3, k_3, h_3 are arbitrary constants, k_3, h_3 are the wave numbers of the x_direction and y_direction respectively, $w_3=-\frac{-5h_3^2+16k_3^6+20k_3^3h_3}{k_3}$ is the wave speed. We can see u_1 is a soliton solution which is shown in Fig. 1.

3.2. Case 2

$$a_3=\frac{a_1^2(k_1^4-h_3^2k_1^2+2k_1^4h_3+2k_1^2h_3-h_1k_1-2h_1h_3k_1-h_1^2-4h_1k_1^3+5k_1^6-k_1^2-3k_1^5h_1+3k_1^8)}{4(k_1^4-k_1^4h_3+3+5k_1^2-h_3^2k_1^2-4k_1^2h_3-3h_3+2h_1k_1^3-h_1^2+2h_1k_1-k_1^6-2h_1h_3k_1)},$$

$$w_1=\frac{10h_3k_1-5h_1+10k_1^3h_3-10h_1h_3+5k_1^3+9k_1^5-5k_1+5h_1k_1^4-k_1^7+5h_1^2k_1-5h_3^2k_1}{(1+k_1^2)},$$

$$w_3=\frac{(-9k_1^2+1+5k_1^4h_3+10h_1h_3k_1-5h_3^2-5h_3-5k_1^4+5k_1^6-10h_1k_1-10k_1^3h_1+5h_1^2)}{(1+k_1^2)},$$

$$a_2=0,\ a_1=a_1, \tag{9}$$