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Study of interpolation methods for high-accuracy computations on overlapping grids

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ABSTRACT

Overset strategy can be an efficient way to keep high-accuracy discretization by decomposing a complex geometry in topologically simple subdomains. Apart from the grid assembly algorithm, the key point of overset technique lies in the interpolation processes which ensure the communications between the overlapping grids. The family of explicit Lagrange and optimized interpolation schemes is studied. The a priori interpolation error is analyzed in the Fourier space, and combined with the error of the chosen discretization to highlight the modification of the numerical error. When high-accuracy algorithms are used an optimization of the interpolation coefficients can enhance the resolvality, which can be useful when high-frequency waves or small turbulent scales need to be supported by a grid. For general curvilinear grids in more than one space dimension, a mapping in a computational space followed by a tensorization of 1-D interpolations is preferred to a direct evaluation of the coefficient in the physical domain. A high-order extension of the isoparametric mapping is accurate and robust since it avoids the inversion of a matrix which may be ill-conditioned. A posteriori error analyses indicate that the interpolation stencil size must be tailored to the accuracy of the discretization scheme. For well discretized wavelengthes, the results show that the choice of a stencil smaller than the stencil of the corresponding finite-difference scheme can be acceptable. Besides the gain of optimization to capture high-frequency phenomena is also underlined. Adding order constraints to the optimization allows an interesting trade-off when a large range of scales is considered. Finally, the ability of the present overset strategy to preserve accuracy is illustrated by the diffraction of an acoustic source by two cylinders, and the generation of acoustic tones in a rotor–stator interaction. Some recommandations are formulated in the closing section.

1. Introduction

Direct Noise Computation (DNC), consisting in solving the acoustic and aerodynamic fields in the same run, increasingly becomes a viable tool for analysis of engineering problems in which noise plays a significant role. In fact, DNC has already been used to study fundamental aspects of noise generation and propagation, such as jets [1,2], or cavities [3,4]. Numerical algorithms minimizing dispersion and dissipation errors are generally required to resolve the weak acoustic wave and preserve their characteristics during long-range propagation [5]. This can be achieved by the use of high-accuracy central difference schemes [6]. Note that a similar constraint is also familiar in the DNS/LES framework, where fine-scale turbulent structures have to be computed on a given grid. Quasi-spectral finite-difference approximations are also widely used for that purpose due to their simplicity and efficiency. The extension to complex geometries of practical interest is however not evident. A first step has been provided by the use of curvilinear grids [7,8] which employs body-fitted grids. This guarantees an accurate treatment of wall boundary conditions, but it is not an easy task to control the grid density: either the grid spacing deteriorates as the distance from the body increases, or the grid is too clustered in some regions, leading to a waste of computational time. Furthermore, the design of multiblock structured meshes with a sufficient regularity is often a challenging task. A solution to control the homogeneity and isotropy of the grid is the use of overset methods. This method consists in solving partial differential equations on different grids which overlap partially. The great interest is thus to decompose a complex domain into several simpler sub-domains, where the high-accuracy schemes can still be used independently. The simple sub-domains are overlapped and interpolations are used to ensure communications.

This method has been first introduced by Benek et al. [9] in the beginning of the 1980s to simulate the flow around a space shuttle. Inspired from the work of Kreiss [10], Chessire and Henshaw [11] studied the generic features of a composite grid builder. Numerous particular cases must be treated such as the overlap of two grids

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near a solid boundary, as explained for instance by Petersson [12]. A free library called *Overture* developed by Henshaw et al. [13] is now available. The next step has been to combine the overlapping grid capability with adaptive mesh refinement techniques, as demonstrated recently by Henshaw and Schwendeman [14], Saunier et al. [15], Sitaraman et al. [16], or Péron et al. [17]. The review by Prewitt et al. [18] shows the maturity of the method for aerodynamic applications with moving grids.

In the context of Computational AeroAcoustics (CAA), high-accuracy algorithms are generally retained, which require relatively regular grids, and large stencils to evaluate derivatives. Yin and Delfs [19] has proposed a first extension of the overset technique with Dispersion Relation Preserving (DRP) schemes [20]. Sherer and Scott [21] have developed the method for compact schemes, and Desquesnes et al. [22] have applied it to a CFD/CAA coupling. Emmert et al. [23,24] used the *Overture* libraries to perform overset simulations with eleven-point stencil finite-difference DRP schemes. When high-order numerical schemes are retained, the interpolation scheme necessary to ensure the communication between the grids should not reduce the global accuracy of the algorithm.

The aim of the present paper is precisely to investigate the interpolation errors in order to choose an interpolation method tailored to the discretization algorithms. The main properties of an interpolation scheme are summarized in a first part. The family of explicit Lagrange or optimized interpolation is detailed in the second section where the extension to multidimensional state space is discussed. A static error analysis based on Fourier representation is proposed in the third section. Sensibly different conclusions can be inferred by the dynamic error analysis of the fourth section. The last section is dedicated to more challenging benchmark cases, such as the diffraction of a source by two cylinders, or the interaction of a gust and a cascade of vanes, representative of the rotor–stator interaction noise. Some recommendations are drawn in the concluding section.

2. Role of interpolations for overset grids

2.1. Principle of the method

The example of Fig. 1 provides an illustration of the principle for two overlapping grids in one space dimension. Two identical regular grids shifted by half a grid spacing are used, but the generalization to any multidimensional grids is straightforward. Grid 1 (on top) ended at index n, and grid 2 (bottom) starts at index 1. An information propagating from left to right is thus known on grid 1, but not on grid 2. Values must be transmitted from grid 1 to grid 2 to sustain the propagation. This communication involves interpolations from interior points (white points) of grid 1 toward interpolated points (black points) of grid 2. In overset terminology, white points are the donor points, whereas black points are receiver points, sometimes referred to as ghost points. The number of ghost points is fixed by the width of the discretization stencil. For instance, an eleven-point centered finite difference scheme is chosen in the example of Fig. 1, so that five ghost are added at the right of grid 1. Likewise five ghost are added at the left of grid 2 to allow a two-way communication. The region between the first interpolation point of grid 2 and the last interpolation point of grid 1 is the overlapping zone. Apart from the direct cost due to interpolation operations, the size of the overlap will add an extra cost. It is thus interesting to keep this zone to a minimum.

For explicit interpolation, the distinction between donor and receiver points imposes a minimal distance between the points, directly linked to the stencil of the interpolation scheme. When a donor point can also be a receiver point, the interpolation is said implicit. That means that at least one of the values used to perform the interpolation is an unknown variable to be interpolated on the other grid. The two-sided interpolation processes are coupled and require the solution of a linear system of equations, which can be expensive [11]. Nevertheless, an implicit interpolation allows a reduced overlapping zone, and becomes pertinent for complex geometries, where the gap between two bodies is small for instance. In the following of the study, only explicit interpolations are discussed. This choice is also motivated by the easier implementation on parallel computers, inevitable when three-dimensional applications on large grids are tackled.

2.2. Properties of an interpolation scheme

Stability issues can arise when considering non-centered stencils [25], or extrapolation [26]. It is then possible to combine an optimization in the wavenumber space and a constraint on the amplification to stabilize the interpolation scheme [26,25]. In the following, only centered interpolations with an even number of stencil points are considered, so that no stability issue arises.

Another issue is the conservative character of the interpolation, which is crucial for transonic or supersonic flows. Conservative interpolation schemes [27–29] are cumbersome for high-order multidimensional applications, so that practitioners prefer the use of non-conservative interpolations, which can be sufficient for weak shocks [30,24], or in conjunction with an adaptive refinement technique to locally increase grid resolution near shocks [31,14]. In the present paper, only subsonic compressible problems are considered.

The main issue in the computational aeroacoustics and large-eddy simulation frameworks is the high-accuracy of the interpolation scheme. Chessire and Henshaw [11] have obtained theoretical results for elliptic problems on composite meshes, and show that

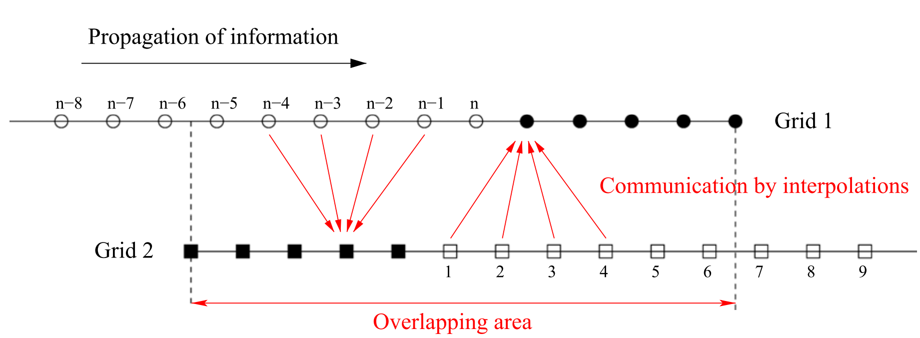

Fig. 1. 1-D example of two overlapping grids. Each grid is both receiver and donor of information. The black points are the interpolation points, called ghost points. The white points are interior points on which derivatives are computed on a centered eleven-point stencil. Thus, five ghost points in the overlapping area are needed. The four arrows symbolize an explicit interpolation from a 4-point stencil.