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Coprime factors reduction methods for linear parameter varying and uncertain systems☆

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Abstract

We present a generalization of the coprime factors model reduction method of Meyer and propose a balanced truncation reduction algorithm for a class of systems containing linear parameter varying and uncertain system models. A complete derivation of coprime factorizations for this class of systems is also given. The reduction method proposed is thus applicable to linear parameter varying and uncertain system realizations that do not satisfy the structured ℓ_2-induced stability constraint required in the standard nonfactored case. Reduction error bounds in the ℓ_2-induced norm of the factorized mapping are given.

Keywords: Coprime factors; Balanced truncation; Linear parameter-varying systems; Uncertain systems

1. Introduction

Model reduction methods with guaranteed error bounds have previously been established for linear fractional and uncertain systems [2,14,37,17,9,22,13,47]. A simplistic view of most of these existing reduction results is as generalizations of the balanced truncation and singular perturbation approximation techniques developed for standard state-space models [23,1,16,20,34,30]. As such, an appropriate generalization of state-space type realizations is typically used to describe the system. In order to apply the reduction methods, the generalized state-space system models are then required to satisfy a *robust* or a *structured* stability condition.

In this paper, we propose a method for the reduction of a class of generalized state-space systems containing linear parameter varying (LPV) and uncertain systems that do not satisfy the structured stability constraints required by the existing methods. In particular, we consider an extension of the coprime factors approach proposed by Meyer for standard state-space systems [29]; a complete derivation of coprime factorizations for this class of systems thus is presented as well. The systems we consider are therefore only required to be stabilizable and detectable in the sense defined by Lu et al. [26]. Error bounds are given in the ℓ_2-induced norm of the factorized linear fractional mapping, where this norm is computed over a unity norm-bounded set. These error bounds are thus useful for stability robustness analysis when interpreted in the robustness framework for coprime factors, or in a gap-metric framework [27,46,19,45].

We begin this paper with a brief overview of the linear fractional framework now commonly used to represent uncertain systems and linear parameter varying systems, and more recently, linear time-varying systems and spatial array systems; for simplicity we will collectively refer to the systems we consider as LFT systems. This overview is followed by an outline of stability conditions for these systems, and a review of existing model reduction results; this material is found in Section 2.

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The proposed factorized reduction method and the development of coprime factorizations for LFT systems is then presented in Section 3, and a basic computational reduction approach is given. Note that in the derivation of our results, we focus on dynamical systems evolving in discrete-time. The results presented herein are based on a preliminary reduction algorithm given in [7].

2. Preliminaries

Matrices in the real and complex numbers will be written $\mathbf{R}^{n\times m}$ and $\mathbf{C}^{n\times m}$, respectively; the $n\times n$ identity matrix is denoted by I_n. For a matrix $A\in\mathbf{C}^{n\times m}$, A^* denotes the complex conjugate transpose, and more generally for operators the adjoint. The dimensions of a matrix A are denoted $\dim(A)$. When a matrix A has only real eigenvalues we will use $\lambda_{\min}(A)$ to indicate the smallest of these. For notational convenience, dimensions will not be given unless pertinent to the discussion.

In this paper $\ell(\mathscr{X})$ denotes the linear space of sequences indexed by $\{0,1,2,\ldots\}$ taking values in the Euclidean space $\mathscr{X}$. The subspace $\ell_2(\mathscr{X})$ contains the sequences that are square summable; it has the usual norm

$$\|x\|_2 := (|x(0)|^2+|x(1)|^2+|x(2)|^2+\cdots)^{1/2},$$

where $|\cdot|$ denotes the Euclidean norm on $\mathscr{X}$. We will often abbreviate these denotations by ℓ and ℓ_2 when the base space $\mathscr{X}$ is clear from the context or not relevant to the discussion.

The vector space of linear mappings on ℓ will be denoted by $\mathscr{L}(\ell)$. Note that $\mathscr{L}(\ell)$ includes maps between spaces with different base spaces, but this is not explicitly represented in our notation. It will be useful in the sequel to refer to the infinite block-matrix associated with a mapping $\mathscr{G}\in\mathscr{L}(\ell)$: we will use the notation $[\mathscr{G}]_{ij}$ to refer to the matrix entries of this representation with respect to the standard basis for ℓ.

Throughout the paper λ will denote the standard shift or delay mapping on ℓ. The *causal* subset $\mathscr{L}_\mathrm{c}(\ell)$ of the linear mappings consists of the operators in $\mathscr{L}(\ell)$ which commute with λ; namely, this set consists of mappings which have lower block-triangular infinite-matrix representations with respect to the standard basis for ℓ. Similarly, we define $\mathscr{L}_\mathrm{c}(\ell_2)$ to be the *bounded* linear mappings on ℓ_2 that are causal. The induced norm of an operator $\mathscr{G}\in\mathscr{L}_\mathrm{c}(\ell_2)$ is given by

$$\|\mathscr{G}\|_{\ell_2\to\ell_2} := \sup_{x\in\ell_2, x\neq 0}\frac{\|\mathscr{G}x\|_2}{\|x\|_2}.$$

Given any element $\mathscr{G}$ in $\mathscr{L}_\mathrm{c}(\ell_2)$ we can extend its domain to all of ℓ using its infinite matrix representation. Thus we can properly regard $\mathscr{L}_\mathrm{c}(\ell_2)$ as a subspace of the vector space $\mathscr{L}_\mathrm{c}(\ell)$. An important property of the subspace $\mathscr{L}_\mathrm{c}(\ell)\subset\mathscr{L}(\ell)$ is that the inverse of any element, if it exists, will also be in $\mathscr{L}_\mathrm{c}(\ell)$; that is, if $\mathscr{G}\in\mathscr{L}_\mathrm{c}(\ell)$ has an inverse in $\mathscr{L}(\ell)$ it must be causal.

Given a matrix A in $\mathbf{R}^{n\times m}$ it clearly defines a memoryless mapping in $\mathscr{L}_\mathrm{c}(\ell_2)$ by pointwise multiplication; in the paper we will not distinguish between this mapping and the matrix A, and will just refer to this memoryless mapping as a “matrix”.

2.1. Linear fractional transformations

The LFT paradigm, described below and pictured in Fig. 1, traditionally has allowed for a mathematical representation of uncertainty in system models.

In the systems we consider, we assume G is a matrix and Δ could represent any of the following: repeated scalar uncertainty structures, exogenous time-varying parameters in linear parameter varying systems, temporal and spatial transform variables in spatial array systems. For specific examples of physical systems leading to these types of models see [5,44,10,11,3,35].

The mapping Δ will be parametrized in a special way in terms of an operator p-tuple denoted by $\overline{\delta}=(\delta_1,\delta_2,\ldots,\delta_p)$, where each δ_i is in $\mathscr{L}_\mathrm{c}(\ell_2(\mathbf{R}))$. Given the p-tuple of dimensions $\overline{m}=(m_1,\ldots,m_p)$ we associate with $\overline{\delta}$ the operator

$$\Delta(\overline{\delta}) = \mathrm{diag}[\delta_1 I_{m_1},\ldots,\delta_p I_{m_p}]. \tag{1}$$

As in Fig. 1 we will often suppress the explicit dependence on $\overline{\delta}$ in our notation. Here the notation $\delta_i I_{m_i}$ is used to signify the operator in $\mathscr{L}_\mathrm{c}(\ell_2(\mathbf{R}^{m_i}))$ whose action on any element $x\in\ell_2(\mathbf{R}^{m_i})$ is defined by

$$(\delta_i I_{m_i})x := (\delta_i x_1, \delta_i x_2,\ldots,\delta_i x_{m_i}),$$

where the scalar sequences x_j are the channels of x; more precisely, $x =: (x_1,x_2,\ldots,x_{m_i})$. Thus Δ is a member of $\mathscr{L}_\mathrm{c}(\ell_2(\mathbf{R}^m))$, where $m := m_1+\cdots+m_p$.