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On the use of Brinkman penalization method for computation of acoustic scattering from complex boundaries

Youngmin Bae [1], Young J. Moon *

Department of Mechanical Engineering, Korea University, Seoul 136-713, South Korea

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ABSTRACT

In this study, a Brinkman penalization method (BPM) is extended for prediction of acoustic scattering from complex geometries. The main idea of the BPM is to model the solid obstacle as a porous material with zero porosity and permeability. With the aim of increasing the spatial accuracy at the immersed boundaries, computation is carried out on the boundary-fitted Cartesian-like grid with a high-order compact scheme combined with one-side differencing/filtering technique at the boundaries, while a slip boundary condition at the wall is imposed by introducing the 'anisotropic' penalization terms to the momentum equations. Several test cases are considered to demonstrate the accuracy, robustness and feasibility of the BPM. Numerical results are in excellent agreement with the analytic solutions for single and two cylinder scattering problems. The present BPM is then used to solve the acoustic scattering from a three-element high-lift wing (30P30N model).

1. Introduction

Numerical simulation of acoustic scattering from complex geometries has received attention in a wide range of aeroacoustic problems, such as slat noise and flap side-edge noise from a multi-element airfoil in high-lift configuration, rotor–stator interaction noise in turbo-machinery, etc. There are two main strategies in direct simulation of acoustic scattering from solid boundaries of complexity, i.e. structured/unstructured body-fitted grid methods [1–3] and immersed boundary methods [4–6]. In the former, implementation of wall boundary condition is straightforward, attaining a desired degree of accuracy at the boundaries. However, when complex geometries are concerned, the structured body-fitted grid method or even the overset grid method [7–9] often meets difficulties associated with the grid generation as well as with the quality of the grids. A discontinuous Galerkin (DG) method [10–12] based on the unstructured grids promises success for real complex geometries but computational cost has always been an issue.

In this regard, an immersed boundary technique can be considered as an alternative because of its simple and efficient implementation for arbitrarily shaped surfaces with reasonable computational cost. Following the pioneering work of Peskin [13], a number of immersed boundary methods have been proposed to handle the complex geometries [4,14–16]. Among them, the Brinkman penalization method (BPM) [16], which was originally developed to model the fluid flow in porous media, appears attractive because of its easiness to handle the solid obstacle by simply treating as a porous medium of high impedance. In BPM, porosity and permeability in the penalty terms which are added in the compressible Navier–Stokes equations are set to zero in the solid region to impose the immersed boundary effect on the fluids. A no-slip boundary condition is therefore enforced naturally at the solid boundary. There are, however, two inherent limitations with this penalization technique. First, it requires a large number of grid points in solid region to retain the order of accuracy at the wall, thus making the method impractical at highly sophisticated geometries. Another drawback is that only the no-slip boundary condition is satisfied at the solid wall, whereas a slip boundary condition has to be met with the full or linearized Euler equations.

In the present study, we address these numerical issues. With aim of increasing the spatial accuracy at the embedded boundary, we conform the immersed boundary grids to the actual shape of the surface following the idea of reshaped cell approach [17,18]. The slip boundary condition at the solid surface is imposed by introducing the 'anisotropic' penalization terms in the momentum equations. The validity of the present method is then assessed by considering the acoustic scattering from (i) a single cylinder, (ii) two circular cylinders and (iii) three element high-lift wing with the deployed slat and flap. We also discuss numerical issues related to the implementation of the reshaped cell approach and to the stiffness due to the penalty terms.

* Corresponding author.
E-mail address: yjmoon@korea.ac.kr (Y.J. Moon).
[1] Present address: Korea Atomic Energy Research Institute, Daejeon 305-353, South Korea.

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2. Computational methodologies

The computational methodologies are given in this section, including the governing equations for the Brinkman penalization method, the high-order numerical schemes with the one-sided differencing/filtering techniques, the implicit treatment of the penalty terms, and the boundary-fitted grid generation details.

2.1. Brinkman penalization method

In many of aeroacoustic problems, a two-dimensional acoustic field around a steady mean flow is governed by the linearized Euler equations written as,

$$\frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial x_j}\left(\rho' U_j + \rho_0 u'_j\right) = 0 \quad (1)$$

$$\frac{\partial u'_i}{\partial t} + \frac{\partial}{\partial x_j}(u'_i U_j) + \frac{\partial p'}{\partial x_i} + \left(u'_j + \frac{\rho'}{\rho_0}U_j\right)\frac{\partial U_i}{\partial x_j} = 0 \quad (2)$$

$$\frac{\partial p'}{\partial t} + \frac{\partial}{\partial x_j}\left(U_j p' + \gamma P u'_j\right) + (\gamma - 1)\left(p'\frac{\partial U_j}{\partial x_j} - u'_j\frac{\partial P}{\partial x_j}\right) = S \quad (3)$$

where S is the source term, ρ_0, U_i and P represent the laminar or turbulent flow field, and ρ', u'_i and p' are the fluctuating variables non-dimensionalized by the ambient density ρ_0, the speed of sound c_0, and the reference pressure $\rho_0 c_0^2$, respectively. In the conventional body-fitted grid method, the acoustic field can be directly solved with Eqs. (1)–(3), while a slip boundary condition is explicitly imposed at the solid conformal boundaries.

In the present Brinkman penalization method, the slip boundary condition at the impermeable surface can be imposed by adding the penalty terms into the momentum equations and modifying the continuity and energy equations as:

$$\frac{\partial \rho'}{\partial t} + \frac{1}{\varepsilon}\left[\frac{\partial}{\partial x_j}\left(\rho' U_j + \rho_0 u'_j\right)\right] = 0 \quad (4)$$

$$\frac{\partial u'_i}{\partial t} + R_{ij}u'_j + \frac{\partial}{\partial x_j}(u'_i U_j) + \frac{\partial p'}{\partial x_i} + \left(u'_j + \frac{\rho'}{\rho_0}U_j\right)\frac{\partial U_i}{\partial x_j} = 0 \quad (5)$$

$$\frac{\partial p'}{\partial t} + \frac{1}{\varepsilon}\left[\frac{\partial}{\partial x_j}\left(U_j p' + \gamma P u'_j\right) + (\gamma - 1)\left(p'\frac{\partial U_j}{\partial x_j} - u'_j\frac{\partial P}{\partial x_j}\right)\right] = S \quad (6)$$

where ϵ is the porosity defined as the ratio of the volume occupied by the fluid to the total volume of porous material, ranging from 0 to 1. In Eq. (5), R_{ij} is the 'anisotropic' permeability tensor [19] that enforces the embedded solid surface to satisfy the slip boundary condition, and is given by:

$$R_{ij} = \frac{1}{K}n_i n_j \quad (7)$$

where K is the non-dimensionalized permeability of homogeneous porous medium, and n_i is the unit normal vector to the impermeable boundary.

The main advantage of this penalization technique is that no additional treatment is needed to impose the boundary condition. Instead, a single set of governing equations with penalty terms is applied to the whole computational domain, in which different porosity ϵ and permeability K are assigned for fluids and solids. In this approach, the only thing needed is to calculate the wall normal vector, $\vec{n}$ inside of the obstacles by the use of level set method or analytically providing the normal vector, and to define appropriate porosity and permeability for each region (see Fig. 1). For example, when porosity is equal to unity and permeability becomes infinite in the fluid region, Eqs. (4)–(6) returns to conventional equations for wave propagation. On the contrary, when porosity and permeability approach zero for solid surfaces, the inertia terms become negligible compared to the penalty terms in the momentum equations so that only the normal component of the velocity is forced to be zero and the slip condition is asymptotically satisfied at the impermeable surfaces. Further discussion of the Brinkman penalization method will be made in the following section.

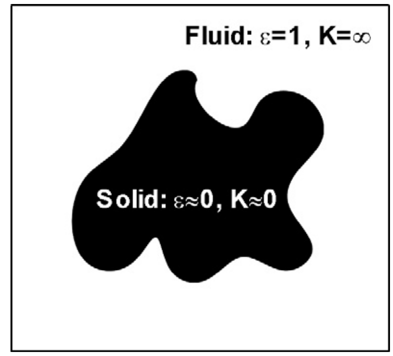

Fig. 1. Concept of Brinkman penalization method.

2.2. Asymptotic analysis

Consider a two-dimensional wave propagation in a fluid at rest ($U_i = 0$). In this case, the governing equations for the Brinkman penalization method (BPM), Eqs. (4)–(6) can be re-written as:

$$\frac{\partial \rho'}{\partial t} + \frac{1}{\varepsilon}\left[\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right] = 0 \quad (8)$$

$$\frac{\partial u'}{\partial t} + \frac{\partial p'}{\partial x} + \frac{n_x^2}{K}u' + \frac{n_x n_y}{K}v' = 0 \quad (9)$$

$$\frac{\partial v'}{\partial t} + \frac{\partial p'}{\partial y} + \frac{n_x n_y}{K}u' + \frac{n_y^2}{K}v' = 0 \quad (10)$$

$$\frac{\partial p'}{\partial t} + \frac{1}{\varepsilon}\left[\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right] = 0 \quad (11)$$

where the hydrodynamic density, $\rho_0 = 1$ and the relation of $\gamma P = \rho_0 c_0^2 = 1$ are used. Multiplying n_x and n_y to Eqs. (9) and (10), respectively, and combining the resultant equations yield,

$$\frac{\partial u'_n}{\partial t} + \frac{\partial p'}{\partial n} + \frac{u'_n}{K} = 0 \quad (12)$$

where $u'_n = n_x u' + n_y v'$ is the wall normal velocity and $\partial p'/\partial n = n_x \partial p'/\partial x + n_y \partial p'/\partial y$ is the normal derivative of pressure fluctuation on the solid surface.

Similarly, taking the dot product of a unit tangent vector $\vec{s}$ with Eqs. (9) and (10) yields,

$$\frac{\partial u'_s}{\partial t} + \frac{\partial p'}{\partial s} = 0 \quad (13)$$

In Eqs. (12) and (13), it is clearly shown 'anisotropic' penalty terms in momentum equations are only responsible for the change of fluctuating variables in the wall normal direction. In other words, only the normal component of fluctuating velocity, u'_n tends to be zero at the embedded boundaries, because in Eq. (12) the first two terms become negligible and the penalty term u'_n/K dominates as $K \to 0$ at the solid surface. This result suggests the slip boundary condition on the solid surface be asymptotically satisfied

$$u'_n \approx 0, \quad \frac{\partial p'}{\partial n} \approx 0 \quad (14)$$

So, the present extension of BPM provides a simple procedure for imposing the immersed boundary effect on the fluid without additional treatment for boundary condition.

Now, in order to verify the physical consistency of the present BPM, the asymptotic analysis is performed for wave