Computers & Fluids 39 (2010) 1585–1596

Contents lists available at ScienceDirect

Computers & Fluids

journal homepage: www.elsevier.com/locate/compfluid

Multi-level adaptive simulation of transient two-phase flow in heterogeneous porous media

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ARTICLE INFO

Article history:
Received 17 November 2009
Received in revised form 6 May 2010
Accepted 17 May 2010
Available online 2 June 2010

Keywords:
Two-phase flow
Heterogeneous porous media
Adaptive mesh refinement
Stabilized finite element method

ABSTRACT

An implicit pressure and explicit saturation (IMPES) finite element method (FEM) incorporating a multi-level shock-type adaptive refinement technique is presented and applied to investigate transient two-phase flow in porous media. Local adaptive mesh refinement is implemented seamlessly with state-of-the-art artificial diffusion stabilization allowing simulations that achieve both high resolution and high accuracy. Two benchmark problems, modelling a single crack and a random porous medium, are used to demonstrate the robustness of the method and illustrate the capabilities of the adaptive refinement technique in resolving the saturation field and the complex interaction (transport phenomena) between two fluids in heterogeneous media.

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1. Introduction

Modelling of two-phase flow in porous media plays a key role in many engineering areas such as environmental remediation [1,2], oil recovery [3–6] and water management in polymer electrolyte fuel cells [7–12]. In polymer electrolyte fuel cells, which motivated in part the developments described herein, water produced at the cathode as a result of the electrochemical reaction can condense [7,8], and is eventually transported through the porous electrode by a combination of mechanisms, including capillary diffusion. At high reaction rates however, an imbalance between liquid water production and transport can result in flooding of the electrode and, consequently, restricted access of the reactant gases to the reactions sites (catalyst layer); this results in a significant performance drop. Understanding of the two-phase transport processes and design of the porous media to mitigate this are therefore crucial and can be facilitated by robust and physically representative simulations. A number of recent publications have addressed some of the modelling challenges associated with two-phase transport in complex porous media. These include the development of improved numerical schemes for simulation of multi-phase, multi-component processes [13]; interface conditions and linearization schemes [14]; advanced numerical procedures based on high-order time integration schemes [15], fractional flow approaches [16], and reduced degrees of freedom [17]. Theoretical investigations based on pore-network models [18], non-oscillation central scheme [19], and multi-scale finite volume/element methods [20–24] have also been developed. Helmig et al. [25] note that numerical methods have to be able to capture both advection or diffusion/dispersion dominated processes. An excellent review of the recent modelling efforts and current challenges is provided by Gerritsen and Durlofsky [5]. A key challenge remains the robust and accurate resolution of fine-scale localized flow.

In transient two-phase flow simulations related to petroleum engineering, the implicit pressure and explicit saturation (IMPES) algorithm, originally developed by Sheldon et al. [26] and Stone and Gardner [27], is widely used. The basic idea of this classical method when applied to two-phase flow in porous media is to separate the computation of pressure from that of saturation. Namely, the coupled system is split into a pressure equation and a saturation equation, and the pressure and saturation equations are solved using implicit and explicit time approximation approaches, respectively. This method is easy to implement and efficient to solve, and requires less memory than other methods such as the simultaneous solution method [28]. Detailed discussions of this method can be found in [26,27], and recent algorithmic improvements are discussed in Chen et al. [29,30].

The numerical simulation of transient two-phase flow transport in heterogeneous porous media (Fig. 1) is computationally expensive, and adequate resolution of complex flow features is not always possible, thus compromising the reliability of the results. Achieving physically representative simulations that resolve all salient length and time scales and localized flow features efficiently remains a challenge. An alternative to global mesh refinement which demands very large computing resources, is adaptive mesh

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doi:10.1016/j.compfluid.2010.05.011

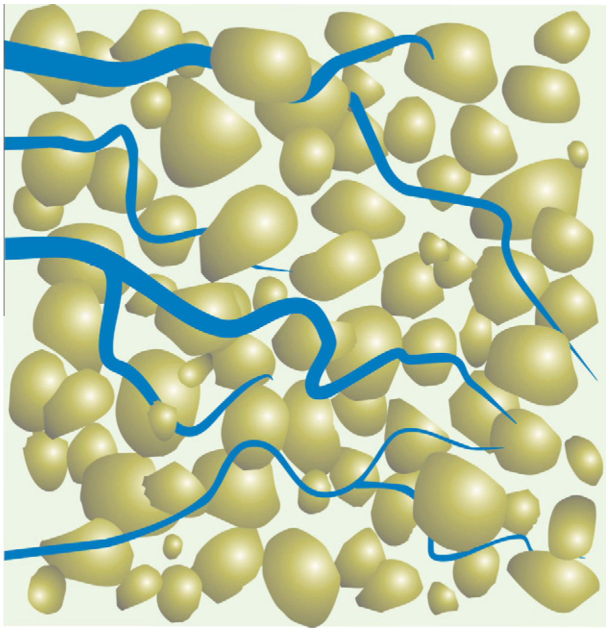

Fig. 1. Schematic of fluid flow in a heterogeneous porous medium.

refinement (AMR). A variety of AMR methods have been proposed depending on the type of physical problem and associated partial differential equations (PDE), and a large body of literature [31–33] exists for these methods. One can use a simple refinement indicator, such as those proposed in [34], to refine and coarsen the mesh at each time step, depending on where the discontinuities (phase boundaries in the present context) propagate. Recent work by Noelle et al. [35] shows that a central scheme with AMR can be implemented on non-conforming 3D Cartesian grids to extend the classical hydrodynamics AMR framework [30]. Smoothness indicators for conservation laws were developed by [36]. Another approach to adaption is the use of a moving-mesh method such as that of Tang and Tang [37] to align the mesh with the important features of the flow. In any case, the major advantages of using grid adaption are high-quality resolution of the physical features as they evolve in space and time while simultaneously reducing computational cost by refining only in areas where necessary and coarsening in areas where unnecessarily fine grids exist. Note that in the context of multi-phase flow the porous medium is frequently strongly heterogeneous within the computational domain. However, as we will show below, there is no need to resolve these heterogeneities everywhere unless they interact with flow fronts. Consequently, adaptive mesh refinement has the potential to significantly reduce the computational cost of multi-phase flow simulations. Despite these obvious advantages, the literature is relatively limited for transient adaptive methods suitable for multi-phase flow in porous media.

When a general continuous finite element discretization is adopted for the saturation transport (advection) equation in two-phase flow problems, spurious and unphysical oscillations appear in the solution, requiring the introduction of a stabilizing (diffusive) term [38]. However, this results in smearing of sharp fronts and can also cause grid-orientation difficulties [38]. Finding the right balance between preserving accuracy and providing stability is therefore of great importance in the numerical solution of conservation laws. In this work, we implement the artificial diffusion terms proposed by Guermond and Pasquetti [39]. This entropy-based nonlinear viscosity provides a powerful approach yielding both accuracy and stability. First, the artificial viscosity term acts only in the vicinity of strong gradients in the saturation and other discontinuities [39]; secondly, the term does not affect the solution in smooth regions; and finally the scheme offers higher order accuracy and stability than simple upwind schemes [39]. In this paper, this approach is combined with an IMPES algorithm and we present an extension of shock-type adaptive refinement to saturation gradients to investigate transient transport phenomena in heterogeneous porous media. The use of this shock-type adaptive refinement technique allows us to provide fine-scale resolution locally and to concentrate numerical efforts near the area where the two-phase interfaces evolve.

2. Basic numerical model

Let us consider the flow of two incompressible, immiscible fluids in a porous media domain $\Omega \subset \mathscr{R}^2$ in which the movement (displacement) of two fluids is dominated by viscous effects and the effects of gravity and capillary pressure are negligible. The two phases are referred to as wetting and non-wetting, and identified by subscripts w and nw, respectively. Thus in a water–oil system (hydrophilic case), water is the wetting and oil the non-wetting phase; in the and air–water system (hydrophobic case), air is the wetting phase and water the non-wetting phase. The mass-averaged velocity with which each of the two phases moves is determined by Darcy's law. It states that the velocity is proportional to the pressure gradient [5]:

$$\mathbf{u}_j = -\frac{k_{rj}(S)}{\mu_j}\mathbf{K}\cdot\nabla p, \tag{1}$$

where $\mathbf{u}_j$ is the velocity of phase $j = w, nw$, $\mathbf{K}$ is the permeability tensor, k_{rj} is the relative permeability of phase j, p is the pressure, and $\boldsymbol{\mu}_j$ is the viscosity of phase j. Finally, S is the saturation of the porous media defined as

$$S = \frac{V_w}{V_w + V_{nw}}, \tag{2}$$

where V_w and V_{nw} are the volume fraction of the wetting and non-wetting phases. In this work, the permeability tensor, $\mathbf{K}$, is a second-order diagonal tensor.

After combining Darcy's law with the mass conservation equation, the following set of equations is obtained [5]:

$$\mathbf{u}_t = -\mathbf{K}\lambda_t(S)\nabla p, \tag{3}$$

$$\nabla\cdot\mathbf{u}_t = q, \tag{4}$$

$$\epsilon\frac{\partial S}{\partial t} + \nabla\cdot(\mathbf{u}_t F(S)) = 0, \tag{5}$$

where λ_t is the total mobility, ϵ is the porosity, F is the fractional flow of the wetting phase, q is a source term, and $\mathbf{u}_t$ is the total velocity. These are given by:

$$\lambda_t(S) = \lambda_w + \lambda_{nw} = \frac{k_{rw}(S)}{\mu_w} + \frac{k_{rnw}(S)}{\mu_{nw}}, \tag{6}$$

$$F(S) = \frac{\lambda_w}{\lambda_t} = \frac{\lambda_w}{\lambda_w + \lambda_{nw}} = \frac{k_{rw}(S)/\mu_w}{k_{rw}(S)/\mu_w + k_{rnw}(S)/\mu_{nw}}, \tag{7}$$

$$\mathbf{u}_t = \mathbf{u}_w + \mathbf{u}_{nw} = -\lambda_t(S)\mathbf{K}\cdot\nabla p. \tag{8}$$

For the sake of simplicity, we consider the case with no source term q. Furthermore the porosity ϵ is set to one as it is essentially a scaling factor that does not affect the qualitative behaviour of Eq. (5). For the purpose of this paper, we will assume the following concrete form for the total mobility λ_t and the fractional flow $F(S)$:

$$\lambda_t(S) = \frac{S^2}{\mu_w} + \frac{(1-S)^2}{\mu_{nw}}, \tag{9}$$

$$F(S) = \frac{S^2}{S^2 + 0.2\cdot(1-S)^2}, \tag{10}$$

where $\mu_w = 0.2$ and $\mu_{nw} = 1$.

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