Chinese Journal of Aeronautics, (2016), 29(4): 952–961

Chinese Society of Aeronautics and Astronautics
& Beihang University

Chinese Journal of Aeronautics

cja@buaa.edu.cn
www.sciencedirect.com

Invariant and energy analysis of an axially retracting beam

Yang Xiaodong [a,*], Liu Ming [a], Zhang Wei [a], Roderick V.N. Melnik [b]

[a] *Beijing Key Laboratory of Nonlinear Vibrations and Strength of Mechanical Engineering, College of Mechanical Engineering, Beijing University of Technology, Beijing 100124, China*
[b] *M2NeT Laboratory, The MS2Discovery Interdisciplinary Research Institute, Wilfrid Laurier University, Waterloo, ON N2L 3C5, Canada*

Received 17 June 2015; revised 5 February 2016; accepted 8 April 2016
Available online 22 June 2016

KEYWORDS

Adiabatic invariants;
Asymptotic analysis;
Retracting beam;
Time-varying systems;
Transient dynamics

Abstract The mechanism of a retracting cantilevered beam has been investigated by the invariant and energy-based analysis. The time-varying parameter partial differential equation governing the transverse vibrations of a beam with retracting motion is derived based on the momentum theorem. The assumed-mode method is used to truncate the governing partial differential equation into a set of ordinary differential equations (ODEs) with time-dependent coefficients. It is found that if the order of truncation is not less than the order of the initial conditions, the assumed-mode method can yield accurate results. The energy transfers among assumed modes are discussed during retraction. The total energy varying with time has been investigated by numerical and analytical methods, and the results have good agreement with each other. For the transverse vibrations of the axially retracting beam, the adiabatic invariant is derived by both the averaging method and the Bessel function method.

1. Introduction

The dynamics of a flexible body such as a slender beam or string whose length changes with time has received a good deal of attention in recent years as one of the examples of the time-varying parameter systems. It leads to a typical model of axial moving system which is important in many areas of applications such as spacecraft antennae, elevator cables,[1] band saw blades,[2] paper sheet processing in high-speed copy machines,[3] and others.

The motion of a thin steel plate that is coiled in high-speed automatic coiling machines is an important application that sometimes leads to a violent vibration. Since the original works in this field,[4] such nonlinear dynamic motion has been formed as the spaghetti problem. To study this problem, Carrier[4] used a linear string theory to solve the corresponding eigenvalue problem with a time-varying boundary condition. Sugiyama et al.[5] presented a modeling method and an experimental procedure for the mechanism analysis of the spaghetti problem. In

* Corresponding author. Tel.: +86 10 67396314.
E-mail addresses: jxdyang@163.com (X. Yang), liuming05.03@163.com (M. Liu), sandyzhang0@yahoo.com (W. Zhang).

Peer review under responsibility of Editorial Committee of CJA.

Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.cja.2016.06.009
1000-9361

their study, the effect of the transport velocity and the clearance were demonstrated, and the cause of a significant increase in the flexible body vibration was discussed from an energy balance viewpoint. The so called spaghetti and reverse spaghetti problems were also studied by Kobayashi and Watanabe.[6] They used a mechanical model and experiments to study the dynamic behavior of a flexible beam that was pulled into and drawn out of a gap in an elastic wall with a constant velocity. Mansfield and Simmonds[7] studied the motion of a sheet of paper in high-speed copy machines as an application of such problem.

The earlier detailed research of the deploying or retracting beam was presented by Tabarrok et al.[8] who derived nonlinear equations of motion of a beam with changing length and presented a closed-form similarity solution and a semi-analytic solution. It is known that the gyroscopic terms can be neglected in the model only for the case of low axial velocity. Tadikonda and Baruh[9] presented the analytical investigation of such a model without the effect of the gyroscopic terms. A finite element model of the axially moving beam based on a geometrically nonlinear beam formulation was studied by Downer and Park,[10] where the varying length of a beam was implemented by applying a moving finite element reference grid. They also formulated the equations of motion using the Hamilton principle. Kalaycioglu and Misra[11] presented approximate analytical solutions which were obtained for the transverse oscillations of deploying or retracting appendages of beam and tether types. Matsuzaki et al.[12] provided experimental data on bending oscillations of a deploying or retrieving beam cantilevered by a clamping device and formulated a finite element analysis for treating the corresponding oscillations. Behdinan and Tabarrok[13] used the updated Lagrangian and the co-rotational finite element methods to obtain the solutions for the geometrically non-linear flexible sliding beam. Tang et al.[14] studied the dynamics of variable-length satellite tethers using a flexible multi-body dynamics method. In their study, the governing equations of the tethers were derived using a new hybrid Eulerian and Lagrangian framework. Tang and Chen[15] investigated the nonlinear free transverse vibration of an in-plane moving plate with constant speed. The governing equation with the boundary conditions was derived from the Hamilton principle and the Hooke's law, and the method of multiple scales was employed to analyze the resulting nonlinear partial differential equation.

Zhu and Ni[16] discussed the energy of vibrations of a translating medium with variable length. Stabilization of a translating medium with variable length requires suppression of both the energy of vibration of a shortening medium and the amplitude of the response of a lengthening medium. Because the boundedness of the displacement did not ensure the boundedness of the energy for a time-varying system, Cooper[17] investigated the dynamic stability from the energy standpoint. Wang et al.[18] analyzed the energy transferred between the transverse vibration and the axial motion, concluding that the material viscosity helped stabilize the transverse vibration in both extension and retraction modes. Chen and Zhao[19,20] proposed a conserved quantity in the studying of axially moving beams and strings. Chen et al.[21] used the energy-like conserved quantity to verify the Layapunov stability of the straight equilibrium configuration of the axially moving material.

The construction of conserved quantity has the potential to reveal the physical interpretation of the non-conservative system. Although the energy-like invariants have been studied in the time-independent parameter systems as shown in Refs.[18–20], the conserved quantity analysis has not been found in the time-varying parameter system. To address the lack of this aspect, the authors discuss the mechanism and the dynamic characteristics of a slender beam that retracts from a prismatic joint. The transfers of energy among different modes are investigated numerically. The construction of adiabatic invariants by the averaging method and the application of the Bessel function method demonstrate the mechanism of the retracting beam. For the first order truncated system, the variation of total energy for transverse vibrations is presented by both analytical and numerical methods.

2. Governing equation

The physical configuration of the retracting beam is given in Fig. 1. The beam is retracted in the prismatic joint under the action of axial force $\boldsymbol{F}$. The moving uniform beam area moment of inertia I, elastic modulus E, mass per unit length m_s, length $L(t)$, and retracting velocity $U(t)$ at time t are used in this research. Viewing the sliding beam as a system of changing mass, one assumes that the part of the beam inside the prismatic joint is non-deformable and has a prescribed axial motion. Along all length of the beam, the axial velocity is uniform since the beam is assumed inextensible. The Euler–Bernoulli beam model is used to determine the transverse motion of the beam described by $Y = Y(X, t)$ in plane as it is retracted from a finite length L_0.

We use three equations, namely, continuity equation, rotational equilibrium equation, and translational equilibrium equation, to derive the governing equation by studying the small segment of the beam as shown in Fig. 2 where $\boldsymbol{M}_1$ and $\boldsymbol{M}_2$ denote bending moment, $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ general force that includes the axial and shear force, and $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ radius vector.

Since the beam is inextensible, the mass per unit length of the projection on the X-axis is

$$m = m_s\sqrt{[1 + (\partial Y/\partial X)^2]} \tag{1}$$

The conservation of mass of a segment of the beam requires

$$\frac{d}{dt}\int_{X_1(t)}^{X_2(t)} m\,dx = U(t)m(X_2, t) - U(t)m(X_1, t) + \int_{X_1(t)}^{X_2(t)} \frac{\partial m}{\partial t}dx = 0 \tag{2}$$

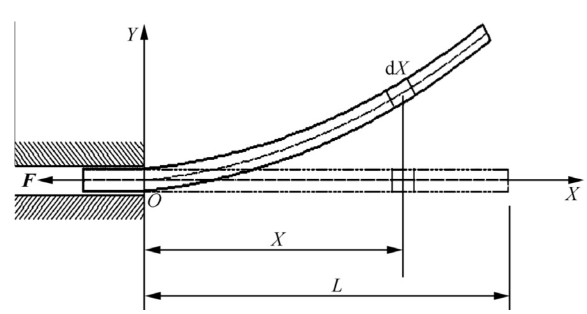

Fig. 1 Model of a retracting cantilever beam.