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Joint calibration algorithm for gain-phase and mutual coupling errors in uniform linear array

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Abstract The effect of gain-phase perturbations and mutual coupling significantly degrades the performance of digital array radar (DAR). This paper investigates array calibration problems in the scenario where the true locations of auxiliary sources deviate from nominal values but the angle intervals are known. A practical algorithm is proposed to jointly calibrate gain-phase errors and mutual coupling errors. Firstly, a simplified model of the distortion matrix is developed based on its special structure in uniform linear array (ULA). Then the model is employed to derive the precise locations of the auxiliary sources by one-dimension search. Finally, the least-squares estimation of the distortion matrix is obtained. The algorithm has the potential of achieving considerable improvement in calibration accuracy due to the reduction of unknown parameters. In addition, the algorithm is feasible for practical applications, since it requires only one auxiliary source with the help of rotation platforms. Simulation results demonstrate the validity, robustness and high performance of the proposed algorithm. Experiments were carried out using an S-band DAR test-bed. The results of measured data show that the proposed algorithm is practical and effective in application.

1. Introduction

Digital array radar (DAR) employs a full digital beamforming (DBF) architecture in the receiving and transmitting system. It has the potential of forming multiple simultaneous beams while providing high anti-interference capabilities. In the last decade, DAR has attracted considerable attention and has been widely used in space surveillance.[1,2] Most array signal processing algorithms, such as DBF and direction of arrival (DOA), rely crucially on the assumption that the array manifold is perfectly known. However, in actual systems, the array manifold is inevitably affected by gain-phase perturbations and mutual coupling effects. As a result, the performance of DAR may be seriously degraded.[3]

Traditional algorithm for array calibration is to carry out measurements using computational electromagnetic solvers,

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which has been applied in some actual radar systems.[4,5] The algorithm has the problem of time consuming and high demands for testing environments. It may be impractical once the array systems are in operation due to the complex electromagnetic environments.

In order to address the problem, a number of self-calibration algorithms that make use of signal processing technology have been developed. Ref.[6] estimates DOA parameters and mutual coupling coefficients using the space alternating generalized expectation maximization algorithm. In Ref.[7], the mutual coupling effects in the uniform linear array (ULA) are inherently eliminated without any calibration sources, but the algorithm requires some extended elements. Refs.[8–10] present a category of algorithms that can iteratively estimate the array manifold errors and the DOAs of impinging signals based on the subspace principle. These calibration algorithms usually suffer from low accuracy, high computational complexity and serious ambiguous problems.

Compared with self-calibration algorithms, active calibration algorithms use auxiliary sources to overcome suboptimal convergence problems and have the potential to achieve better calibration accuracy. The algorithm in Ref.[11] calibrates mutual coupling errors in an arbitrary array using several ideal instrumental elements. But it is difficult to find the ideal elements in practice. A maximum likelihood approach is presented in Refs.[12,13] to estimate the unknown gain-phase, mutual coupling as well as sensor positions. It has the drawbacks of high computation, and the iterations may not be convergent under some conditions. Refs.[14–16] have proposed a category of eigenstructure algorithm that treats gain-phase and mutual coupling errors as a whole. The closed-form of the distortion matrix is derived with the help of some time-disjoint auxiliary sources. The algorithms have been implemented to improve the performance of actual systems.[17,18] But they do not consider about the special structure of distortion matrix and have a strict requirement pertaining to the number of auxiliary sources. The algorithm in Ref.[19] eliminates the repeated entries in the distortion matrix of ULA to reduce the unknown parameters and reaches a better accuracy than the algorithms in Refs.[14,16]. However, the algorithm requires the precise knowledge of locations of auxiliary sources, which may not be available in some actual applications.

In practical situations, it may be difficult to access the precise directions of auxiliary sources. However, it is possible to determine the angle intervals between them using additional equipment, such as rotating platforms. This paper focuses on the problems of joint calibration of gain-phase and mutual coupling errors in the above scenario. The proposed algorithm firstly develops a simplified form of the distortion matrix according to its special structure. Then the relationship between the distortion matrix and the DOAs of calibration sources is derived. Finally one-dimension searching is employed to obtain the angles and the least-square estimation of distortion matrix is also provided. The proposed algorithm achieves high accuracy and behaves robustly when the incident angles of auxiliary sources are not known precisely.

The paper is organized as follows. In Section 2, the signal model of ULA is demonstrated and the problem of array calibration is illustrated. In Section 3, the proposed algorithm for array calibration in the presence of gain-phase errors and mutual coupling errors is developed. Computer simulations and experimental results of measured data are presented and analyzed in Section 4, followed by conclusions in Section 5.

2. Signal model and problem formulation

Consider a ULA consisting of N omnidirectional antenna elements with the space d between neighboring elements. There are M narrowband signals $s_1(t), s_2(t), \ldots, s_M(t)$ located in the far-field region. The signals imping on the ULA from different directions of $\phi_1, \phi_2, \ldots, \phi_M$, with respect to the normal line of the ULA. The signals are incoherent with each other with a wavelength of λ. The additive noise is zero-mean, random process with a variance of σ^2. The outputs of the array can be written as

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \tag{1}$$

where $\boldsymbol{x}(t) = [x_1(t), x_2(t), \ldots, x_N(t)]^{\mathrm{T}}$, $\boldsymbol{s}(t) = [s_1(t), s_2(t), \ldots, s_M(t)]^{\mathrm{T}}$, $\boldsymbol{n}(t) = [n_1(t), n_2(t), \ldots, n_N(t)]^{\mathrm{T}}$ are the output vector, signal vector and noise vector, respectively. $\boldsymbol{A} = [\boldsymbol{a}(\phi_1), \boldsymbol{a}(\phi_2), \ldots, \boldsymbol{a}(\phi_M)]$ is the ideal array manifold matrix, where $\boldsymbol{a}(\phi_i) = [1, \mathrm{e}^{-\mathrm{j}2\pi d \sin\phi_i/\lambda}, \ldots, \mathrm{e}^{-\mathrm{j}2\pi(N-1)d\sin\phi_i/\lambda}]^{\mathrm{T}}$ denotes the ideal steering vector of the i th signal.

Taking gain-phase perturbations and mutual coupling effects into consideration, the outputs can be modified as

$$\boldsymbol{x}(t) = \boldsymbol{C}\boldsymbol{\Gamma}\boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \tag{2}$$

where $\boldsymbol{\Gamma} = \mathrm{diag}(\tau_1, \tau_2, \ldots, \tau_N)$ is a diagonal matrix and τ_i denotes gain-phase errors of the i th element. $\boldsymbol{C} \in \mathbf{C}^{N\times N}$ is the mutual coupling matrix (MCM).

Since the structure of $\boldsymbol{C}$ is highly dependent on the physical structure of the array, it can be considered as a banded symmetric Toeplitz matrix in the case of ULA.[7] Indeed, mutual coupling effects tend to be reciprocal to the distance between elements and may be negligible for the elements separated by a few wavelengths. Therefore, $\boldsymbol{C}$ may be expressed as

$$\begin{cases} \boldsymbol{C}(i,j) = c_{|i-j|+1} & \text{for} \quad i,j = 1,2,\ldots,N \\ 0 < |c_P| < \cdots < |c_2| < |c_1| = 1 \\ c_i = 0 \quad \text{for} \quad i > P \end{cases} \tag{3}$$

where c_i is the mutual coupling coefficient between the first and the ith element. P is the number of non-zero complex coefficients in the first row of the MCM.

The covariance matrix of array output vector is defined as

$$\boldsymbol{R}_{\boldsymbol{x}(t)} = E\{\boldsymbol{x}(t)\boldsymbol{x}^{\mathrm{H}}(t)\} = \boldsymbol{C}\boldsymbol{\Gamma}\boldsymbol{A}\boldsymbol{R}_{\boldsymbol{s}(t)}\boldsymbol{A}^{\mathrm{H}}\boldsymbol{\Gamma}^{\mathrm{H}}\boldsymbol{C}^{\mathrm{H}} + \sigma^2\boldsymbol{I}_N \tag{4}$$

where $\boldsymbol{R}_{\boldsymbol{s}(t)} = E\{\boldsymbol{s}(t)\boldsymbol{s}^{\mathrm{H}}(t)\}$ is the covariance matrix of signals, which is nonsingular when the signals are incoherent. $\boldsymbol{I}_N$ is the $N \times N$ identity matrix.

Performing eigen-decomposition on the output covariance matrix, it can be written as

$$\boldsymbol{R}_{\boldsymbol{s}(t)} = \sum_{n=1}^{M} \varsigma_n \boldsymbol{e}_n \boldsymbol{e}_n^{\mathrm{H}} + \sum_{n=M+1}^{N} \varsigma_n \boldsymbol{e}_n \boldsymbol{e}_n^{\mathrm{H}} = \boldsymbol{U}_{\mathrm{s}}\boldsymbol{\Sigma}_{\mathrm{s}}\boldsymbol{U}_{\mathrm{s}}^{\mathrm{H}} + \boldsymbol{U}_{\mathrm{no}}\boldsymbol{\Sigma}_{\mathrm{no}}\boldsymbol{U}_{\mathrm{no}}^{\mathrm{H}} \tag{5}$$

In Eq. (5), $\varsigma_1 \geqslant \varsigma_2 \geqslant \cdots \geqslant \varsigma_M$ are the M large eigenvalues of $\boldsymbol{R}_{\boldsymbol{x}(t)}$, and $\varsigma_{M+1} = \varsigma_{M+2} = \cdots = \varsigma_N = \sigma^2$ are small eigenvalues. $\boldsymbol{\Sigma}_{\mathrm{s}} = \mathrm{diag}(\varsigma_1, \varsigma_2, \ldots, \varsigma_M)$ and $\boldsymbol{\Sigma}_{\mathrm{no}} = \mathrm{diag}(\varsigma_{M+1}, \varsigma_{M+2}, \ldots, \varsigma_N)$ are diagonal matrices. $\boldsymbol{U}_{\mathrm{s}} = [\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_M] \in \mathbf{C}^{N\times M}$ is composed of the eigenvectors corresponding to the M large eigenvalues, while $\boldsymbol{U}_{\mathrm{no}} = [\boldsymbol{e}_{M+1}, \boldsymbol{e}_{M+2}, \ldots, \boldsymbol{e}_N] \in \mathbf{C}^{N\times(N-M)}$ contains the rest $N - M$ eigenvectors.