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REVIEW ARTICLE

Hypersonic starting flow at high angle of attack

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Abstract Compressible starting flow at small angle of attack (AoA) involves small amplitude waves and time-dependent lift coefficient and has been extensively studied before. In this paper we consider hypersonic starting flow of a two-dimensional flat wing or airfoil at large angle of attack involving strong shock waves. The flow field in some typical regions near the wing is solved analytically. Simple expressions of time-dependent lift evolutions at the initial and final stages are given. Numerical simulations by compuational fluid dynamics are used to verify and complement the theoretical results. It is shown that below the wing there is a straight oblique shock (OSW) wave, a curved shock wave (CSW) and an unsteady horizontal shock wave (USW), and the latter moves perpendicularlly to the wing. The length of these three parts of waves changes with time. The pressure above OSW is larger than that above USW, while across CSW there is a significant drop of the pressure, making the force nearly constant during the initial period of time. When, however, the Mach number is very large, the force coefficient tends to a time-independent constant, proportional to the square of the sine of the angle of attack.

1. Introduction

In a number of flight problems the wing may involve sudden unsteady motion, including sudden acceleration, vertical step motion, or change of angle of attack. The problem to determine the flow and the force for such problems are known as starting flow problem, firstly studied by Wagner[1] and Walker[2] for the case of incompressible flow at small angle of attack (AoA). For supersonic starting flow of a two-dimensional wing at small angle of attack, a linear theory is built by Heaslet and Lomax,[3] who showed that the flow on both sides of the wing is divided into three regions separated by two characteristic lines. The lift was shown to be constant for a short period of time and then increases monotonically to its steady state value following an arcsine type curve. Lomax et al.[4] then extended the linear theory to consider the indicial lift of two and three dimensional wings at both subsonic and supersonic flow speeds. Lengthy expressions are provided for the calculation of the indicial functions which give the time-dependent variation of the lift. For aeroelasticity application, piston theory was developed[5–8] which can be used to calculate the pressure load due to local motion of the body. For convenience of applications, the lift and moment of the wings are expressed as indicial functions for arbitrary motions of the wings.[9–13] Now the indicial lift refers to a time-dependent aerodynamic response to a step change in airfoil motion. For sufficiently

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small motions, the indicial lift function is a linear perturbation of the airfoil downwash distribution from an established steady state.[13]

Starting flow at large angle of attack has been extensively studied for incompressible flow. Graham[14] found that the initial lift is singular (infinite) when the angle of attack is large, due to the rolling up of trailing edge vortex, in contrary to small AoA starting flow for which the initial lift is one half of its steady state value and increases monotonically with time following a curve known as Wagner function.[1] Recently, the long time behavior of lift variation for large angle of attack starting flow has been studied by using experimental measurement,[15] by discrete vortex simulation,[16] or by theory.[17] According to these studies for high angle of attack incompressible starting flow, the lift undergoes a three-phase variation possible repeatable in time[17]: (a) initial lift drops due to Graham singularity, (b) a Wagner type lift increase enhanced by leading edge vortex close to the leading edge, and (c) a lift drop due to a lift-decreasing trailing edge vortex spiral induced by the leading edge vortex convected to the trailing edge.

In spite of the above numerous studies, starting flow at high angle of attack for compressible flow has received few attentions. Bai, Li and Wu[18] performed numerical computation of starting flow at subsonic, transonic and supersonic speeds at high angle of attack. They showed that for subsonic starting flow at large angle of attack, the initial lift is given by the piston theory, which means the Graham singularity of the initial lift is removed by compressibility. Though trailing edge vortex spirals are still observed, the lift coefficient is a decreasing function of the Mach number for sufficiently large time. This is in contrast with our common knowledge that compressibility increases lift for subsonic flow. No details are given for the properties of supersonic and hypersonic starting flow.

In this paper, we study hypersonic starting flow of a two-dimensional wing without thickness but at large angle of attack. We will derive some useful expressions for the pressure coefficients along the wing and the initial and final value of the normal force coefficient. Section 2 is devoted to flow structure and pressure expressions. Section 3 is concerned with the normal force coefficient. Both theory and computational fluid dynamics simulation (for the Euler equations) are used for analysis.

2. Flow structure and pressure coefficient

A two-dimensional wing (of chord length c_{A}) at rest is impulsively set into motion at a constant speed V_∞ with an angle α between the direction of motion and the chordline of the wing. As usual, we use a body fixed frame with the horizontal axis x along the chordline of the wing and y perpendicular to it. Once started, the flow has an initial uniform velocity V_∞ at angle of attack α. Mach waves in the linear case and shock and expansion waves in the nonlinear case are then generated from the surface of the wing and propagate outward, leading to a time-dependent variation of the pressure on the wing. Thus, the force F, normal to the wing for inviscid hypersonic flow, is also time dependent. The density, pressure, sound speed, Mach number, velocity and velocity components are denoted as ρ, p, a, Ma, V and u, v, respectively. The pressure coefficient and normal force coefficient are defined as $C_p = (p - p_\infty)/q_\infty$ and $C_{\mathrm{n}} = F/(q_\infty c_{\mathrm{A}})$. Here $q_\infty = \frac{1}{2}\rho_\infty V_\infty^2$. The specific heat is $\gamma = 1.4$. We use the subscript "∞" to denote free-stream parameters. On the body fixed frame, the free-stream Mach number $Ma_\infty = V_\infty/a_\infty$ satisfies $Ma_\infty \gg 1$. The angle of attack α is large enough to generate nonlinear shock waves and expansion waves, but low enough in theoretical analysis so that the oblique shock wave from the leading edge is attached. As usual, the force coefficient will be expressed as function of the non-dimensional time τ, which is given by

$$\tau = \frac{V_\infty t}{c_A}$$

This non-dimensional time measures the number of chords traveled at the given time t. We first need to recall the linear solution by Heaslet and Lomax.[3]

2.1. Recall of the linear theory

Supersonic starting flow at small angle of attack has been studied long ago by Heaslet and Lomax.[3] The solution on the surface of the wing is divided into three zones labeled I, II and III. The three regions are schematically displayed in Fig. 1. The boundary between regions I and II is $x_{\mathrm{l}} = (u_\infty - a_\infty)t$ and the boundary between regions II and III is $x_{\mathrm{r}} = (u_\infty + a_\infty)t$. We emphasize that it is the horizontal component u_∞ of the velocity that determines these boundaries. In the linear case of Heaslet and Lomax,[3] this component may be regarded as nearly equal to V_∞, so that $Ma_\infty = V_\infty/a_\infty \approx u_\infty/a_\infty$, but for the present nonlinear case with large angle of attack (considered below), u_∞ differs much from V_∞.

Zone I spreads over $0 < x < x_{\mathrm{l}}$ and has the steady-state Ackreret-type solution with the pressure coefficient given by

$$C_{p,\pm}^{(\mathrm{I})} = \pm\frac{2\alpha}{\sqrt{Ma_\infty^2 - 1}} \tag{1}$$

where "+" denotes the lower surface of the wing and "−" denotes its upper surface.

Zone III spreads over $x_{\mathrm{r}} < x < c_{\mathrm{A}}$ and has simple wave solution

$$C_{p,\pm}^{(\mathrm{III})} = \pm\frac{2\alpha}{Ma_\infty} \tag{2}$$

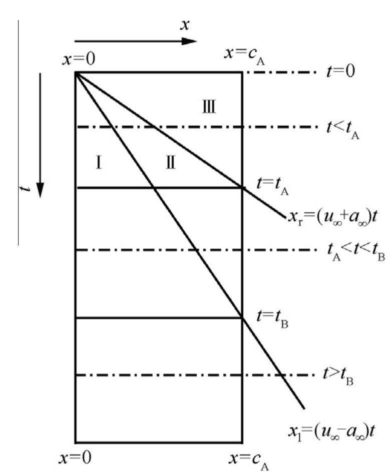

Fig. 1 The flow just below the wing is divided into regions I, II and III.