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Shock wave configurations and reflection hysteresis outside a planar Laval nozzle

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Abstract When the pressure ratio increases from the perfectly expanded condition to the third limited condition in which a normal shock is located on the exit plane, shock wave configurations outside the nozzle can be further assorted as no shock wave on the perfectly expanded condition, weak oblique shock reflection in the regular reflection (RR) pressure ratio condition, shock reflection hysteresis in the dual-solution domain of pressure ratio condition, Mach disk configurations in the Mach reflection (MR) pressure ratio condition, the strong oblique shock wave configurations in the corresponding condition, and a normal shock forms on the exit plane in the third limited condition. Every critical pressure ratio, especially under regular reflection and Mach reflection pressure ratio conditions, is deduced in the paper according to shock wave reflection theory. A hysteresis phenomenon is also theoretically possible in the dual-solution domain. For a planar Laval nozzle with the cross-section area ratio being 5, different critical pressure ratios are counted in these conditions, and numerical simulations are made to demonstrate these various shock wave configurations outside the nozzle. Theoretical analysis and numerical simulations are made to get a more detailed understanding about the shock wave structures outside a Laval nozzle and the RR↔MR transition in the dual-solution domain.

1. Introduction

It is well-known that the aerodynamic study of supersonic jets exhausting from Laval nozzles is of great significance in some engineering applications, especially in space and aeronautical industry. In engineering practice, it is used for the exhaust port of the engines for rockets and supersonic jet airplanes. In rocket engines, nozzles are used to accelerate hot exhaust producing thrust.[1] In many cases the imperfect matching between the ambient pressure and the exit-nozzle pressure leads to complicate shock wave structures. The flow gradually adapts to ambient conditions when it passes through the system of shock waves, so more shock wave structures have been found outside a Laval nozzle from the perfectly expanded condition (the flow is isentropic through the nozzle and supersonic at the nozzle exit without any shock waves or expansion waves) to the third limited condition (a normal shock is located at the exit plane).

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The flow phenomena outside a nozzle[2] have been analyzed in some textbooks and seven conditions of flow regimes were simply referred, which will be given next.

In recent years, great progress has been achieved in investigating substantial aspects of shock wave configurations, especially in the regular reflection (RR)↔Mach reflection (MR) transition. In the transition, both regular reflection and Mach reflection are theoretically possible, and thus this range is considered as the dual-solution domain. The existence of the dual-solution domain led Hornung et al.[3] to predict that a hysteresis can occur in the RR↔MR transition. In the following study, some experimental attempts which were made by Hornung and Robinson[4] to verify this hysteresis failed, thus they believed that the regular reflection wave structures were impossible in the dual-solution domain. According to the principle of minimum entropy production, Li and Ben-Dor[5] analytically showed that the regular reflection wave structure is stable in the dual-solution domain. Soon after that, the hysteresis in the RR↔MR transition was observed experimentally for the first time by Chpoun et al.[6] Then Vuillon et al.[7] did some numerical simulations with the aid of the FCT (Flux-Corrected Transport) algorithm and they illustrated the existence of both regular reflection and Mach reflection wave configurations. After these studies, Shirozu and Nishida[8], Ivanov et al.[9], Hornung[10], and Ben-Dor et al.[11] made some numerical simulations to demonstrate the hysteresis phenomenon.

For a Laval nozzle, the flow structure is similar to the case of supersonic flow between symmetric wedges, so the hysteresis phenomenon can also be observed outside a nozzle. Kudryavtsev et al.[12] and Hadjadj[13] numerically investigated the shock wave reflection transition in a plane overexpanded jet. Euler simulations with a high-order WENO (Weighed Essentially Non-Oscillatory) scheme were carried out and then a second-order TVD (Total Variation Diminishing) scheme is used to conduct Navier–Stokes computations with the k–ε turbulence model. As a consequence, the simulations elucidated the impact of viscosity and turbulence on the shock wave reflection transitions Shimshi et al.[14] performed CFD simulations by solving the steady state Reynolds-averaged Navier–Stokes equations, and a second-order upwind discretization scheme was used in the simulation. As a result, they concluded that the hysteresis phenomenon takes place outside the nozzle even when viscous effects were introduced.

The hysteresis phenomenon has been demonstrated either experimentally or numerically. Together with the hysteresis phenomenon occurred in the RR↔MR transition, a detailed description about more flow structures outside a Laval nozzle will be discussed in the paper. These flow structures contain no shock wave on the perfectly expanded condition, weak oblique shock reflection at regular reflection pressure ratio, shock reflection hysteresis in the dual-solution domain of pressure ratio condition, Mach disk configurations at Mach reflection pressure ratio, the strong oblique shock wave configurations under the corresponding condition and a normal shock forms on the exit plane in the third limited condition.

In the present study, according to the changes of the cross-section area ratio A_e/A^* (the ratio of the exit area to the throat area) and the theory of shock wave reflection, these pressure ratios are under the constraints based on the downstream pressure corresponding to different shock wave reflection phenomena and then they are derived. As a result, the limit curve of pressure ratio under different working conditions for a Laval nozzle is obtained. An appropriate cross-section area ratio is chosen to be applied to the calculation. Therefore, the pressure ratios, which correspond to the perfectly expanded condition, the regular reflection wave configurations condition, the Mach reflection wave configurations condition, strong oblique shock wave configurations condition and the third limited condition in which a normal shock occurs at the exit plane, are computed. The total pressure is set as a certain value, and then the back pressure for every working condition can be gained based on the pressure ratios. Choosing a certain back pressure for each condition to carry out the numerical simulation, the pressure and Mach number contours will be obtained to display the shock wave reflection phenomena.

The hysteresis phenomenon was found by changing parameters for a certain wedge, or it was found in a Laval nozzle. As previously mentioned, the hysteresis phenomenon occurred in the RR↔MR transition, which was found by changing parameters for a certain wedge, or it was found in a nozzle, and it may occur under different back pressures outside a nozzle. The present study is mainly focused on the shock wave structure outside a Laval nozzle. In the dual-solution domain, numerical simulations are conducted with appropriate pressure ratios and then the hysteresis phenomenon can be observed outside the Laval nozzle with a certain cross-section area ratio for the reason that the back pressure changes.

2. Theoretical methods and results

2.1. Theoretical analysis

2.1.1. Flow regimes outside a Laval nozzle

The shock wave configurations outside a nozzle are actual phenomena of shock wave reflection which have been studied for many years. The flow outside a Laval nozzle is different under varying back pressures. The pressure distribution through a nozzle under various back pressures is shown in Fig. 1. In Fig. 1, p is the back pressure, $p*$ represents the back pressure equals the pressure at the throat, and p_0 is the total pressure.

Four flow regimes are possible, for a total of seven conditions, as shown in Fig. 1(a). In Regime I the flow is subsonic throughout a nozzle. The flow rate increases with decreasing back pressure. At Condition f, which forms the dividing line between Regimes I and II, flow at the throat is sonic.

As the back pressure is lowered below Condition f, a normal shock appears downstream from the throat, as shown by Condition e. There is a pressure rise across the shock. Since the flow is subsonic after the shock, the flow decelerates, with an accompanying increase in pressure, through the diverging channel. As the back pressure is lowered further, the shock moves downstream until it appears on the exit plane (Condition d). In Regime II, as in Regime I, the exit flow is subsonic. Since flow properties at the throat are constant for all the conditions in Regime II, the flow rate in Regime II does not vary with back pressure.

In Regime III, as exemplified by Condition c, the back pressure is higher than the exit pressure, but not enough to sustain a normal shock in the exit plane. The flow adjusts to the back pressure through a series of oblique compression shocks outside the nozzle. Condition b represents the perfectly expanded condition. In Regime IV the flow adjusts to the lower back