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Dynamic characteristics of a cyclic-periodic structure with a piezoelectric network

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Abstract This paper deals with a cyclic-periodic structure with a piezoelectric network. In such a system, there is not only mechanical connection but also electrical connection between adjacent periodic sectors. The objective is to learn whether the presence of a piezoelectric network would change the dynamic characteristics of the system. The background of the research is about vibration reduction of a bladed disk in an aero-engine, and the system is simulated by a lumped parameter model. The dynamic equations of the system are derived, and then the analytical solution corresponding to the eigenvalue problem is given. The vibration responses to single traveling wave excitations (EO excitations) and multiple traveling wave excitations (NEO excitations) are studied. The results show that the presence of a piezoelectric network would change the natural frequencies of the system compared with those of the system with the piezoelectric shunt circuit. The forced response is sensitive to the connection type and the elements of the network. An energy analysis of the electro-mechanical coupling system has been performed to understand its dynamic behavior, and the following conclusion is obtained: a vibration reduction to excitations whose primary harmonic component is not zero can be achieved by a parallel piezoelectric network, while a reduction to other excitations should be based on a series piezoelectric network.

1. Introduction

Cyclic-periodic structures are very useful in a lot of engineering equipment, such as satellite antennas and turbomachinery bladed disks. One remarkable dynamic feature of this kind of structure is that the vibration amplitudes of all the sectors are identical, but there is a phase difference in a given mode.[1] However, such structures are more susceptible to excessive vibration because of mistuning, which leads to uncertain durability and reliability. Then it's important to find some ways to

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Nomenclature

N	Number of sectors	γ_{PM}	Non-dimensional intrinsic capacitance of a piezoelectric material
m	Mass of a sector	γ_{SC}	Non-dimensional short-circuit stiffness of a piezoelectric material
c	Damping of a sector	γ_{EM}	Non-dimensional capacitance
k	Stiffness of a sector	$\varepsilon_{\mathrm{RM}}$	Non-dimensional resistance
k_{C}	Coupling stiffness between sectors	ξ	Mechanical damping ratio
I_j	Current ($j = 1, 2, \cdots, N$)	δ	Stationary response
V_j	Voltage ($j = 1, 2, \cdots, N$)	y_j	Non-dimensional displacement ($j = 1, 2, \cdots, N$)
V_0	Voltage in a parallel piezoelectric network	$\boldsymbol{y}$	Non-dimensional displacement vector
R, C_{E}	Resistance and capacitance	q_j	Non-dimensional electrical charge ($j = 1, 2, \cdots, N$)
x_j	Displacement ($j = 1, 2, \cdots, N$)	$\boldsymbol{q}$	Non-dimensional electrical charge vector
f_j	Exciting force ($j = 1, 2, \cdots, N$)	q_0	Non-dimensional electrical charge in a series piezoelectric network
k_{SC}	Short-circuit stiffness of a piezoelectric material	p_j	Non-dimensional excitation ($j = 1, 2, \cdots, N$)
C_{P}	Intrinsic capacitance of a piezoelectric material	α, β	Stiffness coefficients
D_j	Electrical charge ($j = 1, 2, \cdots, N$)	Y_j	Non-dimensional amplitude of displacement ($j = 1, 2, \cdots, N$)
D_0	Electrical charge in a series piezoelectric network	Q_j	Non-dimensional amplitude of electrical charge ($j = 1, 2, \cdots, N$)
η	Electromechanical coupling factor	Q_0	Non-dimensional amplitude of electrical charge in a series piezoelectric network
τ	Non-dimensional time		
ω_{n}	Natural frequency		
ω	Frequency of excitation		
λ	Non-dimensional frequency of excitation		
γ_{C}	Non-dimensional coupling stiffness		

mitigate the vibration and increase their lives. Most of the relevant work has concerned the use of passive devices at the local sector, such as various frictional dampers[2] and viscoelastic damping treatments.[3] In these investigations, the coupling between sectors has not been fully taken into account, nor has the periodicity been exploited in the damping design.

Piezoelectric materials, since they were discovered, have been much developed for vibration control for their electromechanical coupling characteristics. Piezoelectric elements attached on vibrational structures are capable of transforming vibration energy into electrical energy, which can be dissipated or redistributed by the piezoelectric shunt circuit. Two circuit layouts have been studied. One is the separate shunt, and the other is the piezoelectric network.

Research on piezoelectric shunt circuits has proved that the capacitance, the resistance, and the inductance (RLC) in a piezoelectric shunt circuit could be equivalent to additional stiffness, damping, and mass terms respectively in dynamic equations of a mechanical system.[4] Hence, an RLC shunt circuit can perform as a dynamic vibration absorber, but the designed performance is sensitive to the change of excitation frequency and the changes of the system's parameters. Wu[5] demonstrated that if a parallel RL shunt rather than a series RL shunt was chosen, the performance of the system didn't have a significant change but was much less sensitive to changes in the resistive element. Hollkamp[6] proposed a system with the piezoelectric sheet, and the RLC circuits connected with it were parallel while the electrical elements in the circuit were retuned to reduce a modal vibration if a new branch was added. This method is effective when it is applied to a cantilever beam. To obtain an optimal result of controlling vibration, Wu et al.[7–9] proposed another kind of system in which each branch was added a current blocking LC. However, the number of electric circuits increases fast with the growing of the modes number. Behrens and Moheimani[10] studied a way to reduce the shunt circuit's order and obtained very good results. The researches of Kim[11] and Moheimani et al.[12] show that the positions of piezoelectric elements and the reactances of shunted-circuits should be carefully selected to minimize the maximum forced response corresponding to the relevant mode. Zhang et al.[13] used a piezoelectric layered smart plate to reduce vibration of thin-walled structures. It is validated under various excitations. In these researches, it is found that in the low-frequency band, a large inductance is needed to achieve better results, but it is difficult for practical uses, especially for cyclic-periodic structures in rotating.

Compared with works on piezoelectric shunt circuits, few researches have conducted on piezoelectric networks. Several studies have introduced different connected piezoelectric elements to an independent structural component, such as a cantilever beam. The main results indicate that series and parallel connections of piezoelectric layers would cause different constitutive equations of a multi-layer beam.[14–16] The locations of piezoelectric elements could significantly affect the efficiency of passive damping of a structure.[11] Wang et al.[17–22] introduced a piezoelectric network into a mistuned blade disk and got some useful results. In a paper published in 2003,[19] piezoelectric shunt circuits were introduced into a quasi-periodic symmetric structure and connected together with the inductance and capacitance in the circuits. The result shows that there are no localized modes if suitable circuit parameters are selected. In subsequent literature, Wang et al. tried to strengthen the electromechanical coupling with negative capacitance,[20] and focused on the suppression of mistuning response.[21,22] It is indicated that the electromechanical coupling characteristic can effectively suppress vibration localization.