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A fast pulse phase estimation method for X-ray pulsar signals based on epoch folding

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Abstract X-ray pulsar-based navigation (XPNAV) is an attractive method for autonomous deep-space navigation in the future. The pulse phase estimation is a key task in XPNAV and its accuracy directly determines the navigation accuracy. State-of-the-art pulse phase estimation techniques either suffer from poor estimation accuracy, or involve the maximization of generally non-convex object function, thus resulting in a large computational cost. In this paper, a fast pulse phase estimation method based on epoch folding is presented. The statistical properties of the observed profile obtained through epoch folding are developed. Based on this, we recognize the joint probability distribution of the observed profile as the likelihood function and utilize a fast Fourier transform-based procedure to estimate the pulse phase. Computational complexity of the proposed estimator is analyzed as well. Experimental results show that the proposed estimator significantly outperforms the currently used cross-correlation (CC) and nonlinear least squares (NLS) estimators, while significantly reduces the computational complexity compared with NLS and maximum likelihood (ML) estimators.

1. Introduction

Pulsars are highly magnetized, rapidly rotating neutron stars emitting uniquely identifiable signals that are periodical and predictable, throughout the electromagnetic spectrum with periods ranging from milliseconds to thousands of seconds. The repetition period of the radiation signals is simply the rotation period of the neutron star. For some pulsars, the stability of their rotation periods over long timescales is as precise as an atomic clock.[1–3] Of all pulsars, the ones which are visible in the X-ray band of the electromagnetic spectrum are called "X-ray pulsar".[3,4] Compared with the other types of pulsars, the X-ray pulsars are more suitable for use in deep space navigation because of the existence of small size X-ray detectors that can be mounted on a spacecraft.[5]

X-ray pulsar-based navigation (XPNAV) is a developing celestial navigation method and receives increasing attention. It is promising to fulfill completely autonomous navigation to reduce the dependence of current navigation system to ground-based operations, or to argument the current systems

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by employing additional measurements to improve their performance.[4–7] The concept of employing X-ray pulsars to estimate the position of deep space explorer was first proposed in 1974 and has grew rapidly during the last 40 years.[8] United States and the European Space Agency have analyzed the feasibility of XPNAV and continuously studied on the subject.[9–12]

In the recent years, many researchers have investigated different applications of NPNAV, including both absolute and relative navigations.[13–15] It has been shown that one key issue of XPNAV is how to precisely estimate the phase delay between the observed profile and the predicted one.[13–16] To date, many pulse phase estimation algorithms have been developed for XPNAV. The maximum likelihood (ML) phase estimator presented in Ref.[13] directly utilizes the detected photon time of arrivals (TOAs); its estimation accuracy approaches the Cramér-Rao lower bound (CRLB) as the observation duration increases, but the direct employment of the photon TOAs makes the amount of calculation and storage rapidly increase with the growth of received photons and the direct search of the ML solution also leads to a high computational cost. The Fourier transform-based pulse delay estimation method given by Taylor has the advantage that its accuracy is independent of the bin size, but it inevitably involves a straightforward iterative procedure which is computationally intensive.[17] Emadzadeh has proposed two different pulse delay estimators based on the observed profile obtained through epoch folding.[4,13] One uses the cross-correlation (CC) function between the photon intensities and greatly reduces the computational cost compared with the ML estimator. The other is a nonlinear least squares (NLS) estimator. However, both estimators suffer from poor estimation accuracy and are not asymptotically efficient, and the NLS estimator still involves the maximization of a non-convex objective function, thus resulting in a direct search-based procedure. In Ref.[18], the authors describe an approximate ML estimator at low signal to noise ratio (SNR) values. The appropriate approximation of the likelihood function reduces the computational complexity, but results in a degradation of the estimation accuracy as the SNR values increase due to the deviation between the established statistical model of the observed profile and the exact situation. In Ref.[19], the authors recast the problem of pulsar phase estimation as a cyclic shift parameter estimation problem under multinomial distributed observations and develop a fast near-maximum likelihood phase estimation method based on this. This strategy is essentially based on the conditional probability density function of the photon TOA and actually complicates the phase estimation problem of X-ray pulsar, which can be directly mathematically formulated by the epoch folding procedure as presented in the subsequent content of this paper.

In this paper, the X-ray pulsar radiation is characterized as a non-homogeneous Poisson process (NHPP) and the observed profile obtained through the epoch folding procedure is statistically modeled as a heteroscedastic Poisson sequence. Upon this model, we recognize the joint probability distribution of the observed profile as the likelihood function and employ a computationally efficient fast Fourier transform (FFT) based procedure to estimate the pulse phase of X-ray pulsar signals. The rest of this paper is organized as follows. In Section 2, the mathematical models describing the X-ray pulsar signals are presented. Based on this, the heteroscedastic Poisson model formulating the observed profile is established. Section 3 explains how the pulse phase can be estimated by a FFT based procedure by employing the joint probability distribution of the observed profile as the likelihood function. Computational complexity of the proposed estimator is studied in Section 4. In Section 5, experiments are carried out to evaluate this new technique's performance, using both simulated data and real data. Finally, Section 6 concludes the study.

2. Heteroscedastic Poisson model of X-ray pulsar observed profile

The original measurement of X-ray pulsar is the TOAs of all the X-ray photons from the pulsar source as well as the background. TOA of a photon is recorded by the X-ray detector when the photon hits the detecting material.[20–22] In XPNAV, a low power X-ray detection system capable of measuring the photon TOAs with submicro second accuracy is required. The detector must have a low background noise, a large detection area and a light weight. The Naval Research Laboratory (NRL), as part of the Defense Advanced Research Projects Agency (DARPA), has developed a new X-ray silicon-based detector that satisfies all the above-mentioned requirements.[19]

To obtain the observed profile, the measured photon TOAs are first transformed to the solar system barycenter (SSB) and then assembled into a single pulse cycle through the procedure of epoch folding.[3–5,13] In this paper, to focus our discussion, we assume that the photon TOAs have been transformed to SSB. In what follows, according to the presented X-ray detection method, mathematical equations are used to describe the photon TOAs at the SSB; then upon this, statistical properties of the observed profile are given.

2.1. Mathematical model of X-ray pulsar signals

Let N_t be the number of arrival photons at the time interval $(0, t)$. The counting process $\{N(t), t \geqslant 0\}$ can be modeled by a NHPP with a time-varying intensity $\lambda(t) \geqslant 0$.[23,24] For a fixed time interval (t_s, t_e), the number of arrival photons N_{t_s,t_e} is a Poisson random variable with parameter $\int_{t_s}^{t_e} \lambda(t)\mathrm{d}t$. Its distribution low is

$$P(N_{t_s,t_e} = k) = \frac{\left(\int_{t_s}^{t_e} \lambda(t)\mathrm{d}t\right)^k \exp\left(-\int_{t_s}^{t_e} \lambda(t)\mathrm{d}t\right)}{k!} \tag{1}$$

and its mean and variance are

$$E(N_{t_s,t_e}) = \mathrm{var}(N_{t_s,t_e}) = \int_{t_s}^{t_e} \lambda(t)\mathrm{d}t \tag{2}$$

Furthermore, since $\{N(t), t \geqslant 0\}$ has independent increments, the numbers of detected photons in any non-overlapping time intervals are independent from each other. The intensity function $\lambda(t)$ whose unit is ph/s includes all the arriving photons from the X-ray pulsar and the background. It is expressed as

$$\lambda(t) = \alpha + \beta h(\phi(t)) \tag{3}$$

where $h(\phi)$ is the normalized pulsar profile, $\phi(t)$ represents the evolution of the pulse phase with respect to the time t as seen at the SSB, and α and β are the known effective background and source photon arrival rates, respectively. The pulsar profile $h(\phi)$, which is unique to a particular pulsar and defines the