Chinese Journal of Aeronautics, (2014),27(4): 986–993

CSAA

Chinese Society of Aeronautics and Astronautics
& Beihang University

Chinese Journal of Aeronautics

cja@buaa.edu.cn
www.sciencedirect.com

An improved quaternion Gauss–Newton algorithm for attitude determination using magnetometer and accelerometer

Liu Fei, Li Jie *, Wang Haifu, Liu Chang

School of Mechatronical Engineering, Beijing Institute of Technology, Beijing 100081, China

Received 21 August 2013; revised 17 October 2013; accepted 19 December 2013
Available online 24 April 2014

KEYWORDS

Accelerometer;
Attitude determination;
Gauss–Newton algorithm;
Magnetometer;
Measurements

Abstract For the vector attitude determination, the traditional optimal algorithms which are based on quaternion estimator (QUEST) measurement noise model are complicated for just two observations. In our application, the magnetometer and accelerometer are not two comparable kinds of sensors and both are not small field-of-view sensors as well. So in this paper a new unit measurement model is derived. According to the Wahba problem, the optimal weights for each measurement are obtained by the error variance researches. Then an improved quaternion Gauss–Newton method is presented and adopted to acquire attitude. Eventually, simulation results and experimental validation employed to test the proposed method demonstrate the usefulness of the improved algorithm.

1. Introduction

In many spacecraft attitude systems, multi-vector observations are employed to determine attitude via some measurement sensors, including three-axis magnetometers, accelerometers, sun sensors, Earth-horizon sensors, global positioning system (GPS) sensors and star trackers. The specific choice for the onboard sensor hardware is mostly driven by the individual requirements of the spacecraft mission. In many spacecraft attitude determination methods, only two vector measurements are used. For the accuracy and cost requirements of the small unmanned air vehicles (UAVs), a three-axis magnetometer and accelerometer are often adopted to determine attitude to aid gyroscopes.[1] So it is necessary to explore a useful algorithm and conduct comprehensive analysis for the two vector attitude determination method.

The earliest algorithm for determining spacecraft attitude from two vector measurements was three axis attitude determination (TRIAD) algorithm, which has been applied to both ground-based and onboard. However, TRIAD is suboptimal because it ignores one piece of information from one of the unit vector.[2] As most spacecraft are equipped with sensors able to provide surplus measurements and computers in vehicles are able to work at a negligible additional computational cost, optimal algorithms are employed more frequently than deterministic ones. Almost all single-frame algorithms are based on a problem proposed by Wahba.[3] These algorithms,

* Corresponding author. Tel.: +86 10 68912081.
E-mail addresses: lf1985liufei198@bit.edu.cn (F. Liu), lijie@bit.edu.cn (J. Li), lijie@bit.edu.cn (H. Wang), ice417cream@bit.edu.cn (C. Liu).
Peer review under responsibility of Editorial Committee of CJA.

ELSEVIER Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.cja.2014.03.005
1000-9361

which differ from small speed and robustness advantages/disadvantages, include quaternion estimator (QUEST), estimators of the optimal quaternion (ESOQ and ESOQ-2), q-method and singular value decomposition (SVD) method. The basic principles of the above algorithms are to figure out the eigenvector of the maximum eigenvalue.[4]

Mortari et al.[5] proposed an optimal linear attitude estimator (OLAE) and it reformulates the nonlinear constrained problems as a rigorously linear unconstrained problem. But due to the use of the Rodrigues vector, the singularity cannot be avoided. Markley[6,7] particularly researched fast quaternion attitude estimation algorithm and optimal attitude matrix algorithm from two vector measurements based on algorithms mentioned above. In this paper, we adopt the Gauss–Newton method to determine the attitude, because it is more efficient than the previous investigations, particularly for estimation involving large dynamic systems where the computational price to compute the system response and their gradients is high. For aircraft parameter estimation purposes the Gauss–Newton method is therefore widely used.[8] However, most of the researchers employed the Gauss–Newton method with magnetometer and accelerometer measurement vectors to compute the attitude quaternion, while they treated two vectors the same, which is not reasonable for the sensors with different accuracy.[9] In Tanygin's research,[10] the attitude error variance was derived and the weights for each measurement were considered. However, the Gauss–Newton algorithm was just employed to estimate the attitude by GPS antenna baselines only, so it was not essential to normalize the measurements.

The purpose of this paper is to present a corresponding explicit derivation for the unit measurement vector noise form and a variance analysis of the whole algorithm to determine the weights for each measurement. As we all know, these traditional attitude determination algorithms using vector measurements are almost based on the QUEST measurement model which is developed by Shuster[11] for the sensors with comparable accuracy. Meanwhile, the approach made the small field-of-view assumption.[11] Although this measurement model is simple and convenient for the unit-normalization of the measurement vectors, it is not suitable for our application to the two totally different kinds of sensors without small field-of-view.

The structure of the paper is as follows. First, the measurement model for vector sensors is proposed based on first-order Taylor series expansion and new statistical characteristic of measurement noise is derived. After that, the general variance analysis of the algorithm is performed to quantify the approximation error. According to the variance, the weights for two sensors are established. Then, the details of the improved quaternion Gauss–Newton algorithm are discussed. Finally, simulation and experimental tests are conducted to evaluate the whole algorithm.

2. Measurement model

Three-axis magnetometer and accelerometer are both vector sensors. A conventional sensor includes four sources of errors and uncertainties:

(1) Measurement noise.
(2) Measurement bias.
(3) Quantization errors (i.e. analog-to-digital truncation of the measurement).
(4) Sensors misalignment (i.e. angular errors from the mechanical frame non-orthogonal misalignment).[12]

We assume that the two sensors have been calibrated so that correlated errors such as null-shift or Markov biases have been removed. Thus, it is reasonable to assume that the remaining measurement noise term $\Delta\boldsymbol{W}$ can be modeled as a zero-mean Gaussian noise sequence with variance $\boldsymbol{R}$ and the output error model given as

$$\hat{\boldsymbol{W}} = \boldsymbol{W} + \Delta\boldsymbol{W} = \boldsymbol{A}\boldsymbol{V} + \Delta\boldsymbol{W} \tag{1}$$

$$\boldsymbol{R} = E(\Delta\boldsymbol{W}\Delta\boldsymbol{W}^{\mathrm{T}}) = \begin{bmatrix} R_x & 0 & 0 \\ 0 & R_y & 0 \\ 0 & 0 & R_z \end{bmatrix} \tag{2}$$

where $\boldsymbol{W}$ is the true measurement value in the body frame, $\boldsymbol{V}$ is the referenced vector and $\boldsymbol{A}$ is the attitude matrix. In this research, we consider that diagonal elements of the measurement noise variance are not equal after initial calibration in the measured body frame, which is more significant in practice.

The true measurement vector can be reconstructed in unit vector form as

$$\boldsymbol{w} = \boldsymbol{A}\boldsymbol{v} \tag{3}$$

where

$$\boldsymbol{w} = \boldsymbol{W}/|\boldsymbol{W}| \tag{4a}$$

$$\boldsymbol{v} = \boldsymbol{V}/|\boldsymbol{V}| \tag{4b}$$

When measurement noise is present,

$$\hat{\boldsymbol{w}} = \boldsymbol{A}\boldsymbol{v} + \Delta\boldsymbol{w} \tag{5a}$$

$$\hat{\boldsymbol{w}} = \hat{\boldsymbol{W}}/|\hat{\boldsymbol{W}}| \tag{5b}$$

to acquire the variance for the actual unit vector measurement noise, the true vector must be replaced with the measured one in Eq. (1). However, $\boldsymbol{A}\boldsymbol{v}$ could not be separated with the noise in the actual model, so the actual noise model contains nonlinear terms coupled with non-Gaussian component.[13] In order to derive the variance of unit measurement noise $\Delta\boldsymbol{w}$, the new measurement model is obtained based on first-order Taylor series expansion of Eq. (5a), given as

$$\hat{\boldsymbol{w}} \approx \boldsymbol{A}\boldsymbol{v} + \boldsymbol{J}\cdot\Delta\boldsymbol{W} \tag{6}$$

where $\boldsymbol{J}$ is Jacobian matrix of Eq. (5b):

$$\boldsymbol{J} = |\hat{\boldsymbol{W}}|^{-1/2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - |\hat{\boldsymbol{W}}|^{-3/2}\hat{\boldsymbol{W}}\hat{\boldsymbol{W}}^{\mathrm{T}} \tag{7}$$

Meanwhile, the error vector $\Delta\boldsymbol{w}$ lies in the plane perpendicular to $\hat{\boldsymbol{w}}$, characterized by

$$\hat{\boldsymbol{w}} \times \Delta\boldsymbol{w} = \boldsymbol{0} \tag{8}$$

Thus, the new statistical property is approximately Gaussian and is given by

$$E(\Delta\boldsymbol{w}) = \boldsymbol{0}$$

$$\boldsymbol{R}^{\mathrm{unit}} = \boldsymbol{J}\boldsymbol{R}\boldsymbol{J}^{\mathrm{T}} \tag{9}$$