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Dynamic modeling and analysis of vortex filament motion using a novel curve-fitting method

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Abstract Applications of a novel curve-fitting technique are presented to efficiently predict the motion of the vortex filament, which is trailed from a rigid body such as wings and rotors. The governing equations of the motion, when a Lagrangian approach with the present curve-fitting method is applied, can be transformed into an easily solvable form of the system of nonlinear ordinary differential equations. The applicability of Bézier curves, B-spline, and Lagrange interpolating polynomials is investigated. Local Lagrange interpolating polynomials with a shift operator are proposed as the best selection for applications, since it provides superior system characteristics with minimum computing time, compared to other methods. In addition, the Gauss quadrature formula with local refinement strategy has been developed for an accurate prediction of the induced velocity computed with the line integration of the Biot–Savart law. Rotary-wing problems including a vortex ring problem are analyzed to show the efficiency, accuracy, and flexibility in the applications of the proposed method.

1. Introduction

The vortex approach still has large area of applications in aerodynamic load prediction due to its computational efficiency and its wide applicability in fixed-wing aircraft, rotorcraft, wind turbines, and others.[1–7] Helmholtz's theorem of vortex motion and Kevin's circulation theorem describe the basic physics on vortex flows in ideal-fluid. And, Kutta–Joukowski's theorem of lift and Biot–Savart law[8] for induced-velocity field provide practical ways of both modeling and analyzing aerodynamic problems over lifting wings with vortex methods, among which the vortex lattice method (VLM)[1–3] and the free-vortex wake (FVW) method[9–12] combined with the blade element method (BEM) are widely used in many applications until now. These two methods commonly adopt a vortex filament model to meet Helmholtz's theorem and the motion equation of vortex filaments are generally described with Lagrangian approach.[7] The VLM computes vortex strength by applying flow tangency condition at each control point distributed over wing surface and aerodynamic forces by applying Kutta–Joukowski's theorem. Therefore, this method cannot consider the effect of viscous and compressible flow properties. Whereas, the FVW method combined with the BEM computes sectional lift, drag, and pitching moment using airfoil aerodynamic coefficients. It

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utilizes the velocity induced by vortex filaments in computing local angle of attack and Mach number at each section of blade elements. The accuracy of aerodynamic loads predicted with both methods highly depends on the precision in computing both the motion of vortex filaments and the velocity induced by line vortex elements, which consist of main topics of this study.

This paper focuses on the application of curve-fitting techniques[13] in describing time-varying geometry of vortex filaments and in efficiently computing the line integral of Biot–Savart law with enough precision. In this regard, Bliss et al[14] used curved-vortex filaments in estimating an accurate line integration of Biot–Savart law. Their method allows coarser distribution of control points and claimed to reduce computing time without significant loss of accuracy. Celi[15] proposed the use of the method of lines (MOL) to represent the motion equation of vortex filaments with the form of ordinary differential equations (ODEs) and analyzed system characters including system stiffness. His results showed that various ODE solvers can be selectable by considering numerical accuracy and efficiency. Recently, Liu[16] proposed an application of non-uniform rational B-spline (NURBS) curves to represent the geometry of vortex filaments. He transformed the motion equation for vortex filaments into ODEs using a NURBS-based interpolation. His work showed that the computation associated with the calculation of influence coefficients and with time-marching free-wake analysis can be dramatically reduced for fixed-wing cases. Unfortunately, there is no proof of successful implementations of the proposed method for rotary wing problems.

This paper follows the similar approach to Liu's.[16] Various curve-fitting techniques such as Bézier-curves, B-spline,[13] and Lagrange interpolating polynomials[17,18] are investigated to select the best one among them. For this purpose, the dynamic characteristics of ODEs derived using each curve-fitting method are thoroughly analyzed by using the condition number of the mass matrix and the eigenvalues of the system matrix. Thereby, robustness and stability can be identified by analyzing the transformed ODEs. This kind of works has never been done in the previous researches including Liu's study even though these characteristics are crucial in real applications. The natural choice of a line integral algorithm will be a quadrature formula with the curve-fitted geometric model of vortex filaments. However, singular nature in the kernel function of Biot–Savart law can cause difficulties in accurate computation of the induced velocity. A new type of integration strategy is proposed in this paper, where integration points are locally refined with interpolating functions to remove fundamental causes of inaccurate prediction for a line integral.

The proposed methods are applied to time-marching free-wake analyses for rotary wings. Various applications demonstrate the efficiency, accuracy, and flexibility of the proposed method and provide valuable information for the applications of the present approach.

2. Selection of interpolating curves for vortex filaments

Fee-vortex wake methods utilize the Lagrangian description for the motion of a trailed-vortex filament, where the filament geometry is represented by multiple-control points along the vortex-age coordinate $\zeta(t)$ as shown in Fig. 1, where V_∞, $\Gamma(t,\zeta)$, $\boldsymbol{r}(t,\zeta)$, and Ω represent the free-stream velocity, vortex strength, vortex-position vector and rotational speed of the rotor, respectively.

The time-varying geometry of vortex filaments can be described by the motion of each control point governed by the following equation[7] with a local velocity $\boldsymbol{v}$, which is the sum of relative flow velocity and self-induced velocity due to the vortex system:

$$\frac{\mathrm{d}\boldsymbol{r}(t,\zeta)}{\mathrm{d}t} = \boldsymbol{v}(\boldsymbol{r}(t,\zeta)) \tag{1}$$

Since the vortex-age coordinate is a function of time, Eq. (1) can be reduced to Eq. (2):

$$\frac{\mathrm{d}\boldsymbol{r}(t,\zeta)}{\mathrm{d}t} = \frac{\partial\boldsymbol{r}(t,\zeta)}{\partial t} + \frac{\partial\boldsymbol{r}(t,\zeta)}{\partial\zeta}\cdot\frac{\partial\zeta}{\partial t} = \boldsymbol{v}(\boldsymbol{r}(t,\zeta)) \tag{2}$$

The curve-fitting technique in general manner can be expressed by Eq. (3) using temporal position vectors $\boldsymbol{r}_j(t)$ (j = 0,1,...,n) and interpolating curves $\varphi_j(t)$ (j = 0,1,...,n).

$$\boldsymbol{r}(t,\zeta) = \sum_{j=0}^{n}\boldsymbol{r}_j(t)\varphi_j(\zeta) \tag{3}$$

where n represents the total number of control points along the filament and $\boldsymbol{r}_j(t)$ is the representative position vector of jth control point. By substituting Eq. (3) into Eq. (2), the motion equation can be transformed into nonlinear ODEs for $\boldsymbol{r}_j(t)$ as

$$\sum_{j=0}^{n}\dot{\boldsymbol{r}}_j(t)\varphi_j(\zeta) + \dot{\zeta}\sum_{j=0}^{n}\boldsymbol{r}_j(t)\varphi_j'(\zeta) = \boldsymbol{v}(\boldsymbol{r}(t,\zeta)) \tag{4}$$

where $\dot{\boldsymbol{r}}_j(t) = \mathrm{d}\boldsymbol{r}_j(t)/\mathrm{d}t, \varphi_j'(\zeta) = \mathrm{d}\varphi_j(\zeta)/\mathrm{d}\zeta\ (j = 0,1,\cdots,n)$. If the above equations are to be met at arbitrary n-collocation points ζ_i (i = 0,1,...,n) as $\zeta_C = \{\zeta_{\min} = \zeta_0 < \zeta_1 < \cdots < \zeta_{n-1} < \zeta_n = \zeta_{\max}\} - \{\zeta_0\}$, the motion of vortex filament can be represented with the system of n-ODEs as

$$\begin{gathered}\sum_{j=0}^{n}\dot{\boldsymbol{r}}_j(t)\varphi_j(\zeta_i) + \dot{\zeta}\sum_{j=0}^{n}\boldsymbol{r}_j(t)\varphi_j'(\zeta_i) = \boldsymbol{v}(\boldsymbol{r}(t,\zeta_i)) \\ i = 1,2,\cdots,n\end{gathered} \tag{5}$$

The above equations can be rewritten in the matrix form[16] as

$$\boldsymbol{M}\dot{\boldsymbol{r}}(t) = -\dot{\zeta}\boldsymbol{K}\boldsymbol{r}(t) + \boldsymbol{v}(\boldsymbol{r},t) - \boldsymbol{m}_0\dot{\boldsymbol{r}}_0(t) - \dot{\zeta}\boldsymbol{k}_0\boldsymbol{r}_0(t) \tag{6}$$

or

$$\begin{aligned}\dot{\boldsymbol{r}}(t) = &-\dot{\zeta}\boldsymbol{M}^{-1}\boldsymbol{K}\boldsymbol{r}(t) + \boldsymbol{M}^{-1}\boldsymbol{v}(\boldsymbol{r},t) \\ &- \boldsymbol{M}^{-1}\boldsymbol{m}_0\dot{\boldsymbol{r}}_0(t) - \dot{\zeta}\boldsymbol{M}^{-1}\boldsymbol{k}_0\boldsymbol{r}_0(t)\end{aligned} \tag{7}$$

where

$$\begin{cases}\boldsymbol{r}(t) = [\boldsymbol{r}_1(t),\boldsymbol{r}_2(t),\cdots,\boldsymbol{r}_n(t)]^{\mathrm{T}} \\ \boldsymbol{M} = [\varphi_{i,j}] \in R^{n\times n}, \quad \varphi_{i,j} = \varphi_i(\zeta_j), \quad i,j = 1,2,\cdots,n \\ \boldsymbol{K} = [\Delta\varphi_{i,j}] \in R^{n\times n}, \quad \Delta\varphi_{i,j} = \varphi_i'(\zeta_j), \quad i,j = 1,2,\cdots,n \\ \boldsymbol{v}(\boldsymbol{r},t) = [\boldsymbol{v}(\boldsymbol{r},t,\zeta_1),\boldsymbol{v}(\boldsymbol{r},t,\zeta_2),\cdots,v(\boldsymbol{r},t,\zeta_n)]^{\mathrm{T}} \\ \boldsymbol{m}_0 = [\varphi_{0,j}]^{\mathrm{T}}, \quad j = 1,2,\cdots,n \\ \boldsymbol{k}_0 = [\varphi_0'(\zeta_j)]^{\mathrm{T}}, \quad j = 1,2,\cdots,n\end{cases} \tag{8}$$

where $\boldsymbol{M}$ and $\boldsymbol{K}$ are the mass and stiffness matrices of the system, $\boldsymbol{m}_0$ and $\boldsymbol{k}_0$ are system vectors related to vortex release velocity and position, respectively. The point ζ_0 corresponds