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A new method of single celestial-body sun positioning based on theory of mechanisms

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Abstract Considering defects of current single celestial-body positioning methods such as discontinuity and long period, a new sun positioning algorithm is herein put forward. Instead of traditional astronomical spherical trigonometry and celestial coordinate system, the proposed new positioning algorithm is built by theory of mechanisms. Based on previously derived solar vector equations (from a $C^1R^2P^2$ series mechanism), a further global positioning method is developed by inverse kinematics. The longitude and latitude coordinates expressed by Greenwich mean time (GMT) and solar vector in local coordinate system are formulated. Meanwhile, elimination method of multiple solutions, errors of longitude and latitude calculation are given. In addition, this algorithm has been integrated successfully into a mobile phone application to visualize sun positioning process. Results of theoretical verification and smart phone's test demonstrate the validity of presented coordinate's expressions. Precision is shown as equivalent to current works and is acceptable to civil aviation requirement. This new method solves long-period problem in sun sight running fixing and improves applicability of sun positioning. Its methodology can inspire development of new sun positioning device. It would be more applicable to be combined with inertial navigation systems for overcoming discontinuity of celestial navigation systems and accumulative errors of inertial navigation systems.

1. Introduction

Despite a widespread application of global navigation satellite systems (GNSS), celestial positioning has still held an important position no matter as an independent navigation method or as a part of an integrated navigation system.[1–4] It is well accepted that celestial navigation is capable of full autonomy

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and strong anti-interference. To both military aviation and civil aviation, autonomy and anti-interference are important to a navigation system. According to recommendation of next-generation navigation, performance based navigation (PBN) has been proposed by the International Civil Aviation Organization (ICAO). PBN demands an autonomous navigation, including: the warning precision, which is defined as integrity monitoring alert limit (IMAL), is 0.3 n miles at approach phase, 2.0 n miles in air route and 1.0 n miles in terminal area.[5] With this precision, celestial navigation system is a suitable solution or a backup at least.

Among many celestial navigation methods, sun positioning is known as the oldest single celestial-body navigation method. It is also regarded as an important foundation of other methods. As a kind of celestial navigation technology, sun positioning is a reliable way to obtain latitudes and longitudes of observation points. And, it is only based on single star's position and orientation information, such as Polaris and solar vectors. It is mainly used to calculate the longitude and latitude coordinates of observation place by the sun direction in earth surface coordinate, including azimuth angle and elevation angle.[6] In recent years, primary research results of single celestial positioning are as follows. Wang et al.[7] reached the conclusion that the position lines of azimuth difference and height difference can be used for the single celestial positioning. Tang and Yang[8] obtained their position information with a single celestial altitude and azimuth information, which overcomes defects of long period and low precision in sun sight running fixing. Jiang and Yin[9] put forward a new method to render the dynamic scene to simulate celestial fixing principle of sextant in navigation and presented a mathematical model of celestial fixing by sextant. It improved that single celestial navigation is an automatic process. Jia et al.[10] realized sun sight running fixing with computer-aided calculating transcendental equation and accuracy compensation theory. Its method can save time and improve the positioning betimes greatly. Based on sun's azimuth angle and elevation angle, Yuan and Zhang[11] designed a positioning system to achieve a semi-automatic single celestial positioning. Wang et al.[12] analyzed relationship between polarization distribution of skylight and sun orientations and put forward a new mode of celestial navigation based on measuring skylight polarization at two mutually perpendicular orientations. Peng et al.[13] built an automatic calculation system of navigation astronomical orientation by C sharp tool and altitude difference principle, which improved speed and precision of calculation.

It can be seen that many studies focused on improving detection instruments' precision and exploring new technologies, such as the high-precision charge-coupled device (CCD) star sensors and the integrated navigation systems.[2,3] Few studies centered on principle and algorithm of single celestial positioning. Besides, there are problems in current single celestial positioning algorithms. For one thing, we always derive and calculate position information depending on traditional celestial coordinates, which need apparent motion of planets, spherical triangle, and some experience-based error corrections.[6,8] For another, a certain principle error happens because of curves replaced by straight lines in process of height difference calculation.[14] Moreover, if we consider formula errors in single celestial navigation process, positioning error in general moments is far more than the error in transit moment. The ''transit moment limitation'' in traditional calculation results in discontinuous navigation information,[6,15] so the real time positioning can be implemented in this way.

To solve problems among the existing single celestial positioning method, calculation framework of solar positioning is reconstructed by theory of mechanisms. Wang[16] and many researchers testified the dependence of solar vectors on coordinates and time of observation point. Authors have introduced a proven $C^1R^2P^2$ mechanism, whose forward kinematics can be used for calculation of solar vector. In an inverse way, latitude and longitude expressions of observation point in the Greenwich mean time (GMT) can be derived by inverse kinematics of mechanisms[17,18] with a given solar vector. In a word, if time information (year, month, date, hour, minute and second) and its corresponding solar vector in an observation point are known, latitude and longitude of this point are solvable.

Since every variable is related to an explicit joint motion defined by theory of mechanisms, the proposed calculation method is likely to be more comprehendible. Based on those achieved joint motions, a positioning solution is mathematically easy to get by solar vectors. A new method is raised to break through traditional celestial coordinate calculation with astronomical trigonometry, overcome the limitations of sun sight running fixing (obtaining observation's position by observing sun's position at two different moments), achieve continuous positioning at a long time, and improve practicability of single celestial-body positioning applications. In addition, this method utilizes only GMT instead of local time in calculation of latitude and longitude. So it can remove the timing error in radio time service process and improve the positioning precision. To reduce the error of the proposed method, it can be used to combine with inertial navigation equipment and random weighting estimation method.[19]

2. A sun positioning algorithm based on theory of mechanisms

A new calculation model of sun positioning can be constructed with a serial mechanism $C^1R^2P^2$, as shown in Fig. 1. We use five kinematic joints of $C^1R^2P^2$ mechanism to simulate the revolution and rotation of earth to the sun. The rotation joint θ_1 stands for the revolution of earth around sun in the ecliptic plane. The prismatic joint d_2 stands for radius change of earth revolution, corresponding to the elliptical orbit. The rotation joint θ_3 stands for reversing rotation corresponding to revolution angle, in order to maintain constant direction of the axis of earth ($\theta_3 = -\theta_1$). The cylindrical joint can be divided into a prismatic joint d_4 and a rotation joint θ_5, stands for a rotation sleeve along polar axis of the earth. The prismatic joint d_6 stands for radius change on latitude circle of the earth. The joint variables d_4, θ_5 and d_6 are used together to simulate an arbitrary site on earth's surface. d_7 has a very tiny value to permit free moving of joint d_4 along the polar axis. The coordinate $oxyz$ is the earth surface coordinate, in which, x stands for positive west, y positive south, and z zenith. According to authors' previous study, solar vector expressions $\boldsymbol{S}$ in $oxyz$ coordinate can be achieved by product of exponentials formula method[20] or Denavit-Hartenberg method[21] in the form of