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Parallel algorithm and its convergence of spatial domain decomposition of discrete ordinates method for solving radiation heat transfer problem

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Abstract To improve the computational efficiency and hold calculation accuracy at the same time, we study the parallel computation for radiation heat transfer. In this paper, the discrete ordinates method (DOM) and the spatial domain decomposition parallelization (DDP) are combined by message passing interface (MPI) language. The DDP–DOM computation of the radiation heat transfer within the rectangular furnace is described. When the result of DDP–DOM along one-dimensional direction is compared with that along multi-dimensional directions, it is found that the result of the latter one has higher precision without considering the medium scattering. Meanwhile, an in-depth study of the convergence of DDP–DOM for radiation heat transfer is made. Analyzing the cause of the weak convergence, we relate the total number of iteration steps when the convergence is obtained to the number of sub-domains. When we decompose the spatial domain along one-, two- and three-dimensional directions, different linear relationships between the number of total iteration steps and the number of sub-domains will be possessed separately, then several equations are developed to show the relationships. Using the equations, some phenomena in DDP–DOM can be made clear easily. At the same time, the correctness of the equations is verified.

1. Introduction

Radiation heat transfer is an important mode of energy transport in the high-temperature system, which has characteristics of non-gray, directional and full-field. In engineering applications, it often couples with combustion and fluid flow, which makes the calculation of radiation heat transfer an extremely time-consuming and memory-intensive module.[1] For heavy gas turbines and high-performance aircraft engine combustion chamber, radiation heat transfer plays an important role in the evaporation of the fuel, the stability of the combustion, the

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formation of the products of combustion and the heat flux to walls.[2] The combustion control and thermal protection technology need carefully consideration of the coupling of thermal radiation, combustion and fluid flow. Due to the limitations of computer storage and computation speed, it is very difficult to simulate in a single processor when considering the coupling of radiation heat transfer, combustion and fluid flow in combustion chamber with a complex shape.

Nowadays, the parallel computation is one of the effective ways to deal with complex numerical problems, especially in the CFD field. For the extreme non-uniform physical fields (boundary layer area, shock wave and burning core area) of combustion chamber with complex shape, spatial domain decomposition parallelization (DDP) method with multi-sub-domain and multi-scale grids tends to be used.[3] However, the parallel computation technology of radiation heat transfer field generally drops behind.

Presently, the parallel computation is mainly used to solute the neutron transport, for example, the energy decomposition parallel computation method in Ref.[4], the angular decomposition parallelization method in Refs.[5,6] and the spatial DDP method in Refs.[7–10] In the angular decomposition parallelization method, the whole angles are divided into several parts where each part contains a certain number of solid angles, and there are same numbers of processors, with each processor treating a part. When the angle is changed by scattering or reflection case, messages may be exchanged between different processors. In spatial domain decomposition parallelization method, the spatial domain is split into several sub-domains, which match the same number of processors, with each processor performing the calculations for one sub-domain. Because all the sub-domains are not independent, the relationship between sub-domains has to be shown by transmission of information between processors.

Although both the neutron transport and the photon transport are all based on Boltzmann transport equation, the parallel computation for solving radiation heat transfer has been less studied.

In 1999, Coelho and Goncalves used DDP method combined with finite volume method to calculate the medium temperature profile in a furnace;[11] in 2006, Thomas et al. used four parallel algorithms with implicit Monte Carlos method to solve the radiation heat transfer in a rectangular box;[12] in 2008, Goncalve and Dos used two processors to calculate radiation heat transfer and fluid flow synchronously in a combustion chamber, then the coupling relationship between them can be shown by the transmission of information between the two processors.[13]

In order to fit the future demand of integrative coupling simulation of combustion, flow field and heat transfer based on CFD frame in the complex structures of heavy gas turbines and aircraft engines, the parallel computation method for radiative heat transfer is necessary to improve the calculation efficiency of thermal radiation model and its flexibility to CFD technique in the integrative coupling simulation system.

In this paper, the idea of DDP for CFD is introduced, and the DDP are combined with the discrete ordinates method (DOM) under the message passing interface (MPI) language environment to simulate radiation heat transfer field. The spatial domain decomposition scheme, the treatment method of numerical boundary and the parallel computation performance of DDP–DOM are investigated for the radiation heat transfer within the rectangular furnace.

Besides, when the DDP algorithms are used, the convergences become slow all the time,[14–16] which affects the parallel efficiency directly.[17] So an in-depth study of the convergence of DDP–DOM for radiation heat transfer is made in the paper. Analyzing the cause of the weak convergence, we relate the number of iteration steps when the convergence is obtained to the number of sub-domains and find the relationship between them to lay the foundation for future research.

2. DOM of radiation heat transfer

For the absorption, scattering media, the radiation transfer equation can be written as

$$\frac{\mathrm{d}I^m}{\mathrm{d}S} = -\kappa I^m - \sigma_s I^m + \kappa I_B + \frac{\sigma_s}{4\pi}\int_{\Omega^l=4\pi} I^l \Phi^{m,l}\mathrm{d}\Omega^l \tag{1}$$

where I is the radiation intensity, subscript B the black body, S path length, κ the absorption coefficient, σ_s the scattering coefficient, Ω the solid angle, and $\Phi^{m,l} = \Phi(\Omega^m, \Omega^l)$ the scattering phase function; m and l are ordinates of radiation intensity.

For opaque, diffuse emission, diffuse scattering boundary wall (subscript "wall" is wall surface), the boundary conditions are:

$$I^m_{\mathrm{wall}} = \varepsilon_{\mathrm{wall}} I_{\mathrm{B,wall}} + \frac{1-\varepsilon_{\mathrm{wall}}}{\pi}\int_{\cos\theta<0} I^m_{\mathrm{wall}}\cos\theta \mathrm{d}\Omega \tag{2}$$

where $\varepsilon_{\mathrm{wall}}$ is the surface emissivity and θ the angle between ordinate m and the normal of the wall.

When all the radiation intensity in different directions has been calculated, the medium temperature can be calculated by Eq. (3):

$$T = \left(\frac{\kappa\int_{\Omega^l=4\pi} I^l\mathrm{d}\Omega^l + q}{4\kappa\sigma}\right)^{\frac{1}{4}} \tag{3}$$

where T is the medium temperature, $\sigma = 5.67\times10^{-8}$ Boltzmann constant, and q internal heat source.

For three-dimensional Cartesian coordinate system (x,y,z) and for structural grids, the discrete ordinates approximation to Eq. (1) can be expressed as

$$\xi^m\frac{\partial I^m}{\partial x} + \eta^m\frac{\partial I^m}{\partial y} + \mu^m\frac{\partial I^m}{\partial z} = -\kappa I^m - \sigma_s I^m + \kappa I_B + \frac{\sigma_s}{4\pi}\sum_{l=1}^{N\Omega} I^m\Phi^{m,l}\omega^l \tag{4}$$

where ξ^m, η^m, μ^m are the ordinate cosines in x, y, z directions and ω^l is weight function in ordinate l.

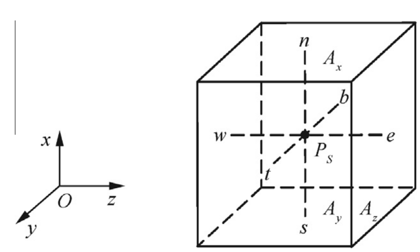

Fig. 1 Schematic of typical control volume.