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Lifetime prediction based on Gamma processes from accelerated degradation data

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Abstract Accelerated degradation test is a useful approach to predict the product lifetime at the normal use stress level, especially for highly reliable products. Two kinds of the lifetime prediction based on Gamma processes were studied. One was to predict the lifetime of the population from accelerated degradation data, and the other was to predict the lifetime of an individual by taking the accelerated degradation data as prior information. For an extensive application, the Gamma process with a time transformation and random effects was considered. A novel contribution is that a deducing method for determining the relationships between the shape and scale parameters of Gamma processes and accelerated stresses was presented. When predicting the lifetime of an individual, Bayesian inference methods were adopted to improve the prediction accuracy, in which the conjugate prior distribution and the non-conjugate prior distribution of random parameters were studied. The conjugate prior distribution only considers the random effect of the scale parameter while the non-conjugate prior distribution considers the random effects of both the scale and shape parameter. The application and usefulness of the proposed method was demonstrated by the accelerated degradation data of carbon-film resistors.

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1. Introduction

With the development of engineering and science technology, more and more products with long lifetime and high reliability have emerged. Thus, accelerated degradation test (ADT), which can provide degradation data in a more timely fashion, has become an appealing approach in reliability assessment and lifetime prediction. An effective ADT must guarantee that the failure mechanism of products under different accelerated stresses remains consistent, so the lifetime characteristics at the normal use stress level can be extrapolated from accelerated degradation data. The applications of ADT can be referred by light bars,[1] integrated logic family,[2] light-emitting diodes[3,4] and carbon-film resistors,[5] etc.

However, the accuracy of lifetime prediction greatly depends on modeling accelerated degradation data (ADD) well or not. With the feature of reflecting the random influences of the internal and external environments, the

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stochastic process models are naturally applicable to modeling degradation data. Among them, the Wiener process has been widely studied by Whitmore and Schenkelberg,[6] Padgett and Tomlinson,[7] Wang,[8] Si et al.[9–11] and Peng and Tseng.[12] As far as the Gamma process, it also attracts great enthusiasm in the field of modeling degradation data after it was firstly used as a degradation model by Abdel-Hamee.[13] Park and Padgett[14] proposed an approach of approximating a Gamma distribution by an inverse Gaussian distribution. Lawless and Crowder[15] and van Noortwijk[16] studied a Gamma process with covariates and random effect. van Noortwijk[16] provided a comprehensive introduction for Gamma processes and surveyed their applications in maintenance. Pan and Balakrishnan[17] used Gamma processes to model the degradation data with multiple performance characteristics. Tsai et al.[18] analyzed and evaluated the effects of misspecification of Gamma and Wiener processes. Besides, there are lots of researches on the optimal design of ADT based on Gamma processes, see Liao and Tseng,[19] Tsai et al.[20]

When modeling ADD, an inevitable work is to specify which parameter of a degradation model should change with accelerated stresses varying and which parameter should remain unchanged. However, it is intractable to determine the relationships between parameters and accelerated stresses since there is no feasible deducting method. Instead, at present the relationships are specified by some assumptions which are made from testing data or engineering experience. For the Gamma degradation model, there are two different assumptions. The most widely applied assumption is that the shape parameter should change with accelerated stresses varying while the scale parameter should remain unchanged, see Park and Padgett,[14,21] Tseng et al.[22] The contrary assumption deemed that the shape parameter should remain unchanged with accelerated stresses varying while the scale parameter should change, see Lawless and Crowder,[15] Wang.[23] In the paper, according to acceleration factor constant hypothesis proposed by Zhou et al.,[24] the relationships between the parameters of the Gamma process and accelerated stresses were mathematically deduced.

Two kinds of the lifetime prediction based on Gamma processes were studied. One is to predict the lifetime of the population from accelerated degradation data, by which the reliability and quality of product can be assessed. The other is to predict the lifetime of an individual by taking the accelerated degradation data as prior information, which can provide decision support for prognostics and health management (PHM). It is well known that the conjugate prior distribution of a Gamma distribution only considers the random effect of the shape parameter. Thus, we also studied a non-conjugate prior distribution which allows for the random effects of both the shape parameter and the scale parameter.

The remainder of this paper is organized as follows. In Section 2, we studied the lifetime prediction of the population. Firstly, the lifetime prediction model based on a Gamma process was introduced, and then the relationships between the parameters of a Gamma process and accelerated stresses were deduced, lastly the method of estimating parameters was also illustrated in the section. In Section 3, we discussed the lifetime prediction of an individual based on Bayesian inference with a conjugate prior distribution and a non-conjugate prior distribution. In Section 4 we provided a case study to illustrate the application and usefulness of the proposed methods. Some conclusions were drawn in Section 5.

2. Lifetime prediction of the population

2.1. Lifetime prediction model

Gamma process is a stochastic process which is applicable to modeling the always positive and strictly increasing degradation data. In mathematics, Gamma process $\{Y(t); Y(0)=0\}$ has independent, non-negative increments $\Delta Y(t) = Y(t+\Delta t) - Y(t)$ that follow a Gamma distribution as

$$\Delta Y(t) \sim \mathrm{Ga}(\alpha(\Lambda(t+\Delta t) - \Lambda(t)), \beta) \tag{1}$$

where $\beta(\beta > 0)$ is a scale parameter, $\alpha(\alpha > 0)$ is a shape parameter and $\Lambda(t)$ is a monotone increasing function of time t with $\Lambda(0) = 0$. According to the additivity of a Gamma distribution, it can be deduced that $Y(t)$ should follow the Gamma distribution $\mathrm{Ga}(\alpha\Lambda(t), \beta)$. The probability density function (PDF) of $Y(t)$ is expressed by

$$f(Y) = \frac{\beta^{\alpha\Lambda(t)}}{\Gamma(\alpha\Lambda(t))} Y^{\alpha\Lambda(t)-1} \exp(-Y\beta) \tag{2}$$

From Eq. (2), the mean of $Y(t)$ is obtained as $\alpha\Lambda(t)/\beta$ and the variance is obtained as $\alpha\Lambda(t)/\beta^2$. Suppose that $y(t)$ is a degradation process and the lifetime ξ is defined as the first passage time of $y(t)$ reaches the failure threshold l. Let y_0 denote the initial value of $y(t)$, then $y(t) - y_0$ follows the distribution $\mathrm{Ga}(\alpha\Lambda(t), \beta)$. Thus, the reliability function can be given by

$$\begin{aligned} P(\xi > t) &= P(y(t) < l) = P(y(t) - y_0 < l - y_0) \\ &= \int_0^{l-y_0} \frac{\beta^{\alpha\Lambda(t)}}{\Gamma(\alpha\Lambda(t))} y^{\alpha\Lambda(t)-1} \exp(-y\beta)\mathrm{d}y \\ &= \frac{1}{\Gamma(\alpha\Lambda(t))} \int_0^{l_\beta} x^{\alpha\Lambda(t)-1} \exp(-x)\mathrm{d}x \end{aligned} \tag{3}$$

where $l_\beta = (l - y_0)\beta$, $x = y\beta$. Substitute the incomplete Gamma function $\Gamma(a,z) = \int_z^\infty x^{a-1}\exp(-x)\mathrm{d}x$ into Eq. (3), then the cumulative distribution function (CDF) of ξ be obtained as

$$F(t) = \frac{\Gamma(\alpha\Lambda(t), l_\beta)}{\Gamma(\alpha\Lambda(t))} \tag{4}$$

From Eq. (4), the exact PDF of ξ for a Gamma degradation model can be extrapolated, but the exact PDF is too complex to practical applications. For mathematical convenience, Park and Padgett[14] provided an approach that a form of Birnbaum–Saunders (BS) distribution was used to approximate the CDF of ξ. With a time transformation $z = \Lambda(t)$, the CDF can be expressed as

$$F_{\mathrm{BS}}(z) \approx \Phi\left[\frac{1}{a}\left(\sqrt{\frac{z}{b}} - \sqrt{\frac{b}{z}}\right)\right] \tag{5}$$

where $a = 1/\sqrt{l_\beta}$ and $b = l_\beta/\alpha$. The PDF can be expressed as

$$f_{\mathrm{BS}}(z) = \frac{1}{2\sqrt{2}ab}\left[\left(\frac{b}{z}\right)^{1/2} + \left(\frac{b}{z}\right)^{3/2}\right] \exp\left[-\frac{(b-z)^2}{2a^2bz}\right] \tag{6}$$

As described by van Noortwijk,[16] empirical studies show that the expected deterioration of $y(t)$ is often proportional to a power law function of time as $E(y(t)) = \alpha\Lambda(t)/\beta =$

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