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Attitude synchronization for multiple spacecraft with input constraints

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Abstract The attitude synchronization problem for multiple spacecraft with input constraints is investigated in this paper. Two distributed control laws are presented and analyzed. First, by introducing bounded function, a distributed asymptotically stable control law is proposed. Such a control scheme can guarantee attitude synchronization and the control inputs of each spacecraft can be *a priori* bounded regardless of the number of its neighbors. Then, based on graph theory, homogeneous method, and Lyapunov stability theory, a distributed finite-time control law is designed. Rigorous proof shows that attitude synchronization of multiple spacecraft can be achieved in finite time, and the control scheme satisfies input saturation requirement. Finally, numerical simulations are presented to demonstrate the effectiveness and feasibility of the proposed schemes.

1. Introduction

The attitude synchronization problem for multiple spacecraft or rigid bodies has attracted considerable attention in recent years. In particular, the use of graph theory which was actively applied in linear multi-agent systems with single and double integrator dynamics produced many interesting results (see Refs. [1–8]). In these papers, attitude synchronization for multiple spacecraft in the presence of modeling uncertainties, external disturbance or communication delays can be guaranteed. However, input saturation problem in the control of spacecraft system has not been considered.

When control input saturation occurs, it can cause the system dynamic's poor performance and even the instability of the system.[9] In Ref. [10], three distributed control algorithms were given for attitude synchronization, the first of which reduced the required control torque by introducing bounded functions. In Refs. [11,12], the velocity-free attitude synchronization control schemes were proposed for multiple spacecraft which could bounded control input. Authors of Ref. [13] studied the attitude synchronization problem of multiple rigid bodies in the presence of communication delay, and showed that a natural saturation was achieved. In those papers that account for actuator saturation problems, the upper bound condition of input of the proposed control schemes require the numbers of neighbors of each spacecraft as *a priori*. However, this could introduce difficulties in tuning the control gains especially in the case that the maximum allowed input values are small and the number of neighbors of each agent may be large. In

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Ref. [14], the synchronization problem of networked Lagrangian systems was addressed and the control input was designed to be *a priori* bounded independently from the information flow in the network.

Most of the existing attitude synchronization control algorithms for multiple spacecraft were asymptotic results, which meant the attitude synchronization could not be achieved in finite time. For theoretical and practical reasons, finite-time control algorithms are more desirable.[15,16] Finite-time control algorithms for a single spacecraft and multiple spacecraft have been developed in Refs. [17–19] and Refs. [20–22], respectively. The authors of Ref. [21] studied the finite-time attitude synchronization problem for multiple spacecraft with considering external disturbances. In Ref. [22], a dynamical synchronization error constructed by the relative translation and rotation between two spacecraft was first introduced and then the terminal sliding mode control laws were designed such that synchronization error can converge to the desired trajectory in finite time. To the best of our knowledge, few results on the finite-time attitude synchronization for multiple spacecraft with input constraints are available in the existing literature.

The main purpose of this paper is to study the attitude synchronization problems for multiple spacecraft with input constraints. Briefly, the contributions of this paper are twofold. First, two distributed control laws are proposed to achieve attitude synchronization asymptotically and in finite time, and particularly, the finite-time control law is the major result of this part. Second, the aforementioned control algorithms allow to generate control inputs which are bounded as *a priori*. Particularly, the upper bound of control input is independent from the number of neighbors of each spacecraft.

The organization of this paper is presented as follows. Preliminaries are introduced in Section 2. Section 3 first investigates the asymptotical attitude synchronization, then studies the finite-time attitude synchronization, for multiple spacecraft with input constraints. In Section 4, simulation results are given and discussed, followed by the conclusions in Section 5.

2. Preliminaries

2.1. Notations

Given a vector $\boldsymbol{\upsilon} = [\upsilon_1\ \upsilon_2\ \upsilon_3]^{\mathrm{T}}$ and $\alpha \in \mathbf{R}$, define $\tanh(\boldsymbol{\upsilon}) = [\tanh(\upsilon_1)\ \tanh(\upsilon_2)\ \tanh(\upsilon_3)]^{\mathrm{T}}$, $\int_0^{\boldsymbol{\upsilon}} \boldsymbol{\tau}\mathrm{d}\boldsymbol{\tau} = \left[\int_0^{\upsilon_1} \boldsymbol{\tau}\mathrm{d}\boldsymbol{\tau} \int_0^{\upsilon_2} \boldsymbol{\tau}\mathrm{d}\boldsymbol{\tau} \int_0^{\upsilon_3} \boldsymbol{\tau}\mathrm{d}\boldsymbol{\tau}\right]^{\mathrm{T}}$, $\mathrm{sig}(\boldsymbol{\upsilon})^{\alpha} = [|\upsilon_1|^{\alpha}\mathrm{sgn}(\upsilon_1)|\upsilon_2|^{\alpha}\mathrm{sgn}(\upsilon_2)|\upsilon_3|^{\alpha}\mathrm{sgn}(\upsilon_3)]^{\mathrm{T}}$, $o(\boldsymbol{\upsilon}) = [o(\upsilon_1)\ o(\upsilon_2)\ o(\upsilon_3)]^{\mathrm{T}}$, where $o(\cdot)$ denotes the infinitesimal of higher order. Moreover, $\|\boldsymbol{\upsilon}\|$ denotes the 2-norm of $\boldsymbol{\upsilon}$.

2.2. Mathematical model of rigid spacecraft

The attitude kinematics and dynamics equations of the ith spacecraft are given as

$$\dot{\boldsymbol{\sigma}}_i = \boldsymbol{H}(\boldsymbol{\sigma}_i)\boldsymbol{\omega}_i \tag{1}$$

$$\boldsymbol{J}_i\dot{\boldsymbol{\omega}}_i + \boldsymbol{\omega}_i^{\times}\boldsymbol{J}_i\boldsymbol{\omega}_i = \boldsymbol{u}_i \tag{2}$$

where $\boldsymbol{\omega}_i \in \mathbf{R}^3$ is the angular velocity of the ith spacecraft with respect to the inertial frame expressed in the body frame of the ith spacecraft; $\boldsymbol{J}_i \in \mathbf{R}^{3\times 3}$ and $\boldsymbol{u}_i \in \mathbf{R}^3$ are the inertia tensor and the control torque of the ith spacecraft, respectively. $\boldsymbol{\sigma}_i \in \mathbf{R}^3$ is the modified Rodrigues parameters (MRP) denoting the rotation from the body frame of the ith spacecraft to the inertial frame. The notation $\boldsymbol{\omega}_i^{\times}$ denotes the cross-product operator of $\boldsymbol{\omega}_i$. The matrix $\boldsymbol{H}(\boldsymbol{\sigma}_i)$ is given by $\boldsymbol{H}(\boldsymbol{\sigma}_i) = \frac{1}{2}\left(\frac{1-\boldsymbol{\sigma}_i^{\mathrm{T}}\boldsymbol{\sigma}_i}{2}\boldsymbol{i}_3 + \boldsymbol{\sigma}_i^{\times} + \boldsymbol{\sigma}_i\boldsymbol{\sigma}_i^{\mathrm{T}}\right)$.

Remark 1. This particular MRP set goes singular when a complete revolution is performed. As is shown in Ref. [23], original MRP vector $\boldsymbol{\sigma}_i$ and its corresponding shadow counterpart $\boldsymbol{\sigma}_i^* = -\boldsymbol{\sigma}_i/(\boldsymbol{\sigma}_i^{\mathrm{T}}\boldsymbol{\sigma}_i)$ could be used to represent spacecraft attitude rotation to avoid the singularity problem. Eqs. (1) and (2) can be expressed by Euler–Lagrange formulation as

$$\boldsymbol{M}_i(\boldsymbol{\sigma}_i)\ddot{\boldsymbol{\sigma}}_i + \boldsymbol{C}_i(\boldsymbol{\sigma}_i, \dot{\boldsymbol{\sigma}}_i)\dot{\boldsymbol{\sigma}}_i = \boldsymbol{u}_i^* \tag{3}$$

where $\boldsymbol{M}_i(\boldsymbol{\sigma}_i) = \boldsymbol{F}_i^{\mathrm{T}}\boldsymbol{J}_i\boldsymbol{F}_i$, $\boldsymbol{C}_i(\boldsymbol{\sigma}_i, \dot{\boldsymbol{\sigma}}_i) = -\boldsymbol{F}_i^{\mathrm{T}}\boldsymbol{J}_i\boldsymbol{F}_i\dot{\boldsymbol{H}}(\boldsymbol{\sigma}_i)\boldsymbol{F}_i - \boldsymbol{F}_i^{\mathrm{T}}(\boldsymbol{J}_i\boldsymbol{F}_i\dot{\boldsymbol{\sigma}}_i)^{\times}\boldsymbol{F}_i$, $\boldsymbol{u}_i^* = \boldsymbol{F}_i^{\mathrm{T}}\boldsymbol{u}_i$, $\boldsymbol{F}_i = \boldsymbol{F}(\boldsymbol{\sigma}_i) = \boldsymbol{H}^{-1}(\boldsymbol{\sigma}_i)$.

Eq. (3) exhibits the following properties.

Property 1. *Matrix $\boldsymbol{M}_i(\boldsymbol{\sigma}_i)$ is symmetric and positive definite.*

Property 2. *Matrix $\dot{\boldsymbol{M}}_i(\boldsymbol{\sigma}_i) - 2\boldsymbol{C}_i(\boldsymbol{\sigma}_i, \dot{\boldsymbol{\sigma}}_i)$ is skew-symmetric.*

2.3. Graph theory

Graph theory is applied to modelling the communication topology among spacecraft. A graph G consists of a node set $V = \{1,2,\cdots,n\}$, an edge set $E \subseteq V\times V$, and a weighted adjacency matrix $\boldsymbol{A} = [a_{ij}] \in \mathbf{R}^{n\times n}$ with weight elements $a_{ij} > 0$ if $(j,i) \in E$, and $a_{ij} = 0$ if otherwise. An edge (i,j) denotes node j can obtain information from node i. Here node i is a neighbor of node j. Graph G is undirected if for any edge $(i,j) \in E$, we have $(j,i) \in E$. A path from node i to node j is a sequence of edges in a graph. The graph is called connected if there is a path between every pair of nodes in a graph. Here nodes are exemplified as a formation of spacecraft.

The Laplacian matrix $\boldsymbol{L} = [l_{ij}] \in \mathbf{R}^{n\times n}$ is defined as $l_{ii} = \sum_{j=1,j\neq i}^{n} a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$. For undirected graphs, both $\boldsymbol{A}$ and $\boldsymbol{L}$ are symmetric.

2.4. Control objective

Our control objectives to be achieved in this paper are stated as follows.

OBJ1: To design a distributed control law for system (1) and (2) such that attitude synchronization can be achieved asymptotically, i.e., $\boldsymbol{\sigma}_i \rightarrow \boldsymbol{\sigma}_j$, $\boldsymbol{\omega}_i \rightarrow \mathbf{0}$, as $t \rightarrow \infty$.

OBJ2: To design a distributed control law for system (1) and (2) such that attitude synchronization can be achieved in finite time, i.e., $\boldsymbol{\sigma}_i \rightarrow \boldsymbol{\sigma}_j$, $\boldsymbol{\omega}_i \rightarrow \mathbf{0}$ in finite time.

In this work we assume that all spacecraft are subject to input saturation constraints such that $\|\boldsymbol{u}_i\| \leqslant u_{\mathrm{M}i}$.

3. Main results

3.1. Control law design for OBJ1

In this section, we consider the attitude synchronization problem for multiple spacecraft with input constraints. The distributed control law for the ith spacecraft is proposed as