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Gray bootstrap method for estimating frequency-varying random vibration signals with small samples

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Abstract During environment testing, the estimation of random vibration signals (RVS) is an important technique for the airborne platform safety and reliability. However, the available methods including extreme value envelope method (EVEM), statistical tolerances method (STM) and improved statistical tolerance method (ISTM) require large samples and typical probability distribution. Moreover, the frequency-varying characteristic of RVS is usually not taken into account. Gray bootstrap method (GBM) is proposed to solve the problem of estimating frequency-varying RVS with small samples. Firstly, the estimated indexes are obtained including the estimated interval, the estimated uncertainty, the estimated value, the estimated error and estimated reliability. In addition, GBM is applied to estimating the single flight testing of certain aircraft. At last, in order to evaluate the estimated performance, GBM is compared with bootstrap method (BM) and gray method (GM) in testing analysis. The result shows that GBM has superiority for estimating dynamic signals with small samples and estimated reliability is proved to be 100% at the given confidence level.

1. Introduction

The description of random vibration signals (RVS) is divided into time domain and frequency domain. Frequency domain analysis method based on power spectral density (PSD) is widely used at present. In environment testing, the estimation of RVS is needed for developing vibration stress conditions and assessing local structure fatigue life. Therefore, the estimated authenticity and accuracy are important guarantee

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for the safety and reliability of airborne platform equipment.[1–4] However, the available methods including the extreme value envelope method (EVEM), the statistical tolerances method (STM) and the improved statistical tolerance method (ISTM) require large samples and typical probability distribution. Moreover, the frequency-varying characteristic of RVS is usually not taken into account.[5–8]

In practical, due to the small number of available sorties, RVS with small samples could be obtained in the stage of flight testing. It is difficult to get a good result if the estimated methods based on large samples and typical probability distribution are still employed to treat this kind of small samples.[9,10] In addition, every element of RVS is a frequency-varying function. Hence, the uncertainty evaluation of RVS is a dynamic process which changes with measured value series. However, the guide to the expression of uncertainty in measurement (GUM) is only suitable for static field. Thus, neither Type A evaluation nor Type B evaluation in GUM can be used for estimating dynamic signals with small samples.[11–13]

Among available estimated methods with small samples, the grey system theory (GST) and the bootstrap method (BM) are two prevailing methods which are widely used in practical engineering.[14,15] For instance, In Ref.[16], a smooth grey reference line obtained by grey dynamic filtering was proposed to estimate surface roughness. In Ref.[17], bootstrap hypothesis was used for testing three-dimensional labeled landmark data. However, the grey model gray method GM(1,1) cannot evaluate uncertainty at the given confidence level;[18] BM generates the additional uncertainty by imitation resampling operation, which causes accuracy loss of the Monte Carlo approximation.[19] According the deficiencies of GST and BM, grey bootstrap method (GBM) is proposed by combining information prediction of GST and probability distribution imitation of BM.[20,21] Therefore, GBM can evaluate the uncertainty without any prior information about probability distribution of random variables.[22] For example, In Ref.[20], GBM was proposed to evaluate uncertainty in the process of dynamic measurement with poor information, and computer simulation and experiment were used to make sure of adaptability of GBM. In Ref.[21], GBM was employed for the reliability analysis of very few failure data with a known or unknown probability distribution. In Ref.[23], a novel poor information Brinell hardness measurement error prediction method was presented which is based on GBM.

In this paper, GBM is proposed for estimating frequency-varying RVS with small samples. Firstly, gray bootstrap modeling and estimated indexes are obtained. Secondly, GBM is applied to estimating RVS of fore cabin and rear cabin in the single flight testing of certain aircraft. In addition, in order to evaluate the estimated performance, GBM is compared with BM and GM in testing analysis.

2. Gray bootstrap modeling

During environment testing, suppose frequency-varying RVS with small samples is represented as a vector $\boldsymbol{X}$ is given by

$$\boldsymbol{X} = [x(f); f = 1, 2, \cdots, F] \tag{1}$$

where $x(f)$ is the measured value of RVS at frequency f, F the number of frequency.

In practice, the vector $\boldsymbol{X}$ can be written by

$$\boldsymbol{X} = [x(f) + c; f = 1, 2, \cdots, F] \tag{2}$$

where according to GM(1,1)[13,22], c is a constant which should make $x(f) + c \geqslant 0$. If $x(f) \geqslant 0$,then let $c = 0$.

The former m data adjacent frequency f is picked out from $\boldsymbol{X}$, and the subsequence vector $\boldsymbol{X}_m$ could be given by

$$\begin{aligned} &\boldsymbol{X}_m = [x_m(u)] \\ &(u = f - m + 1, f - m + 2, \cdots, f;\ f \geqslant m) \end{aligned} \tag{3}$$

where m is the former m data adjacent frequency f, namely bootstrap assessment factor. According to GM(1,1), the smaller the value of the parameter m is, the fresher the information is. The minimum value of the parameter m is 4.

According to BM,[13,22] one data can be obtained by an equiprobable resampling with replacement from Eq. (3), namely imitation resampling operation. An imitation sample containing m data can be obtained by repeating m times. Then B bootstrap samples can be obtained by repeating B times which is given by

$$\boldsymbol{Y}_{\text{Bootstrap}} = [\,\boldsymbol{Y}_1 \quad \boldsymbol{Y}_2 \quad \cdots \quad \boldsymbol{Y}_b \cdots \quad \boldsymbol{Y}_B\,] \tag{4}$$

where $\boldsymbol{Y}_b$ is the bth imitation sample, B the number of the imitation resampling operation. In general, the parameter B affects the estimated performance; the less the value of the parameter B is, the less credible the estimated performance would be.

The BM is a prevalent method for generation of many data and imitation of the unknown probability distribution with small samples. Furthermore, the estimated performance is guaranteed via equiprobable resampling operation from measured value of RVS.

$$\boldsymbol{Y}_b = [y_b(u)] \quad (b = 1, 2, \ldots, B) \tag{5}$$

where $y_b\,(u)$ is the uth sample within $\boldsymbol{Y}_b$.

According to GM(1,1), the accumulated generating operation (AGO) of $\boldsymbol{Y}_b$ is defined by

$$\boldsymbol{X}_b = [x_b(u)] = \left\{ \sum_{j=f-m+1}^{u} y_b(j) \right\} \tag{6}$$

The series vector generated by mean value is given by

$$\begin{aligned} &\boldsymbol{Z}_b = [z_b(u)] = [0.5x_b(u) + 0.5x_b(u-1)] \\ &(u = f - m + 2, f - m + 3, \cdots, f) \end{aligned} \tag{7}$$

In the initial condition $x_b(f - m + 1) = y_b(f-m + 1)$, least square solution (LSS) is given by

$$\begin{aligned} &\hat{x}_b(j+1) = [y_b(f - m + 1) - c_2/c_1]\mathrm{e}^{-c_1 j} + c_2/c_1 \\ &(j = f - 1, f) \end{aligned} \tag{8}$$

where the coefficients, c_1 ($c_1 \neq 0$) and c_2 are given by

$$[(c_1, c_2)]^{\mathrm{T}} = (\boldsymbol{D}^{\mathrm{T}}\boldsymbol{D})^{-1}\boldsymbol{D}^{\mathrm{T}}(\boldsymbol{Y}_b)^{\mathrm{T}} \tag{9}$$

$$\boldsymbol{D} = (-\boldsymbol{Z}_b, \boldsymbol{I})^{\mathrm{T}} \tag{10}$$

$$\boldsymbol{I} = [1 \quad 1 \quad \cdots \quad 1] \tag{11}$$

According to the inverse accumulated generating operation (IAGO) of GM(1,1), the instantaneous value of frequency $w = f + 1$ can be given by

$$\begin{aligned} &\hat{y}_b(w) = \hat{x}_b(w) - \hat{x}_b(w-1) - c \\ &(w = f + 1) \end{aligned} \tag{12}$$

Therefore, the B data at frequency $w = f + 1$ can be obtained constituting a vector as

$$\begin{aligned} &\widehat{\boldsymbol{X}}_w = [\hat{y}_b(w)] \\ &(b = 1, 2, \cdots, B;\ w = f + 1) \end{aligned} \tag{13}$$