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An improved multiple model GM-PHD filter for maneuvering target tracking

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Abstract In this paper, an improved implementation of multiple model Gaussian mixture probability hypothesis density (MM-GM-PHD) filter is proposed. For maneuvering target tracking, based on joint distribution, the existing MM-GM-PHD filter is relatively complex. To simplify the filter, model conditioned distribution and model probability are used in the improved MM-GM-PHD filter. In the algorithm, every Gaussian components describing existing, birth and spawned targets are estimated by multiple model method. The final results of the Gaussian components are the fusion of multiple model estimations. The algorithm does not need to compute the joint PHD distribution and has a simpler computation procedure. Compared with single model GM-PHD, the algorithm gives more accurate estimation on the number and state of the targets. Compared with the existing MM-GM-PHD algorithm, it saves computation time by more than 30%. Moreover, it also outperforms the interacting multiple model joint probabilistic data association (IMMJPDA) filter in a relatively dense clutter environment.

1. Introduction

Multiple target tracking (MTT) is an important theoretical and practical problem, which has been widely applied to military fields such as ballistic missile defense, air reconnaissance and early-warning, battlefield surveillance, etc. and some civil fields such as intelligent vehicle system, air traffic control, traffic navigation and robot vision system, etc. In the MTT problem, the number of targets changes due to targets' appearing and disappearing and it is not known the corresponding relationship between targets and measurements. The probability hypothesis density (PHD) is a novel approach to multi-target multi-sensor tracking. Based on random finite set (RFS) theory, the PHD is the first moment of a point process of a random track set, and it can be propagated by Bayesian prediction and observation equations to form a multi-target, multi-sensor tracking filter. PHD filter provides a straightforward method of estimating the number of targets in the region under observation,[1,2] which has been widely used recently, such as visual tracking,[3,4] track management[5,6] and maneuvering target tracking.[7–9] Sequential Monte Carlo method proposes an implementation of PHD filter.[10,11] The main drawbacks of the approach are the large number of particles

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and the unreliability of clustering techniques for extracting state estimates. Based on Gaussian sum theory,[12] Vo and Ma[13] proposed a closed-form solution to the PHD filter called Gaussian mixture PHD (GM-PHD) filter, which is a solution for multi-target tracking with linear Gaussian models without the need for measurement-to-track data association.

For the problem of tracking highly maneuvering target, it is usually more difficult for the uncertainty of the targets' motion mode. The advantage of PHD is that it can deal with unknown number of targets. But for maneuvering target, it does not have special good method. By using joint PHD distribution, a GM-PHD filter for jump Markov system is proposed in Ref. [7] which can be used in the maneuvering target tracking. In this paper, an improved MM-GM-PHD filter is proposed. Different from the implementation in Ref. [7] using the conditioned model distribution and model probability, the procedure of MM-GM-PHD filter is simplified.

2. Gaussian mixture PHD filter

In the RFS theory, the state of a target is represented by a state vector $\boldsymbol{x}$ and a state set of multiple targets is represented as a random finite set $\boldsymbol{X} = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_{n_x}\}$. Measurement of a sensor is represented by a measurement vector $\boldsymbol{z}$ and the measurement set at that time is also represented as a random finite set $\boldsymbol{Z} = \{\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_{n_z}\}$.

Based on RFS theory, the PHD filter consists of two steps which are prediction and update.[5] The prediction step is

$$\begin{aligned} &D_{k|k-1}(\boldsymbol{x}_k|\boldsymbol{Z}_{1:k-1}) \\ &\quad = \gamma_k(\boldsymbol{x}_k) + \int \varphi_{k|k-1}(\boldsymbol{x}_k, \boldsymbol{x}_{k-1}) D_{k-1|k-1}(\boldsymbol{x}_{k-1}|\boldsymbol{Z}_{1:k-1}) \mathrm{d}\boldsymbol{x}_{k-1} \end{aligned} \quad (1)$$

where $\gamma_k(\boldsymbol{x}_k)$ denotes the intensity function of the random finite set of the new born targets and

$$\begin{aligned} \varphi_{k|k-1}(\boldsymbol{x}_k, \boldsymbol{x}_{k-1}) &= b_{k|k-1}(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}) \\ &\quad + e_{k|k-1}(\boldsymbol{x}_{k-1}) f_{k|k-1}(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}) \end{aligned} \quad (2)$$

where $b_{k|k-1}(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$ denotes the intensity function of the random set of targets spawned from the previous state $\boldsymbol{x}_{k-1}$, $e_{k|k-1}(\boldsymbol{x}_{k-1})$ the probability that the target still exists at time k, and $f_{k|k-1}(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$ the transition probability density of individual targets.

The update step is

$$\begin{aligned} D_{k|k}(\boldsymbol{x}_k|\boldsymbol{Z}_{1:k}) &= D_{k|k-1}(\boldsymbol{x}_k|\boldsymbol{Z}_{1:k-1}) \Bigg[1 - p_{\mathrm{D},k}(\boldsymbol{x}_k) \\ &\quad + \sum_{z_k \in \boldsymbol{Z}_k} \frac{\psi_{k,z_k}(\boldsymbol{x}_k)}{\kappa_k(\boldsymbol{z}_k) + \int \psi_{k,z_k}(\boldsymbol{x}_k) D_{k|k-1}(\boldsymbol{x}_k|\boldsymbol{Z}_{1|k-1}) \mathrm{d}\boldsymbol{x}_k} \Bigg] \end{aligned} \quad (3)$$

where $\psi_{k,z_k}(\boldsymbol{x}_k) = p_{\mathrm{D},k}(\boldsymbol{x}_k) g_{k|k}(\boldsymbol{z}_k|\boldsymbol{x}_k)$ with $p_{\mathrm{D},k}(\boldsymbol{x}_k)$ denoting the probability of detection, $g_{k|k}(\boldsymbol{z}_k|\boldsymbol{x}_k)$ the likelihood of individual targets. $\kappa_k(\boldsymbol{z}_k) = \lambda_k c_k(\boldsymbol{z}_k)$, where λ_k the average number of clutter points per scan, and $c_k(\boldsymbol{z}_k)$ the probability distribution of each clutter point.

The closed form version of the PHD filter for linear Gaussian target dynamics was developed to provide a multi-target tracker without the complexity of the particle PHD filter approach.[7] In the GM-PHD filter, some assumptions are required:

- **A1** Each target evolves and generates observations independently of one another.
- **A2** Clutter is Poisson and independent of target originated measurements.
- **A3** The predicted multi-target random finite set is Poisson.
- **A4** Each target follows a linear Gaussian dynamical model and the sensor has a linear Gaussian measurement model

$$f_{k|k-1}(\boldsymbol{x}|\zeta) = \mathcal{N}(\boldsymbol{x}; \boldsymbol{F}_{k-1}\zeta, \boldsymbol{Q}_{k-1}) \quad (4)$$

$$g_k(z|\boldsymbol{x}) = \mathcal{N}(z; \boldsymbol{H}_k\boldsymbol{x}, \boldsymbol{R}_k) \quad (5)$$

where $\boldsymbol{x}$ and $\boldsymbol{\zeta}$ are state variables, $\mathcal{N}(\cdot; m, P)$ denotes a Gaussian density with mean m and covariance P, $\boldsymbol{F}_{k-1}$ the state transition matrix, $\boldsymbol{Q}_{k-1}$ the process noise covariance, $\boldsymbol{H}_k$ the observation matrix and $\boldsymbol{R}_k$ the observation noise covariance.

- **A5** The survival and detection probabilities are state independent,

$$p_{\mathrm{S},k}(\boldsymbol{x}) = p_{\mathrm{S},k}$$

$$p_{\mathrm{D},k}(\boldsymbol{x}) = p_{\mathrm{D},k}$$

- **A6** The intensities of the birth and spawn random finite sets are Gaussian mixtures of the form

$$\gamma_k(\boldsymbol{x}) = \sum_{i=1}^{J_{\gamma,k}} w_{\gamma,k}^{(i)} \mathcal{N}(\boldsymbol{x}; \boldsymbol{m}_{\gamma,k}^{(i)}, \boldsymbol{P}_{\gamma,k}^{(i)}) \quad (6)$$

$$\beta_{k|k-1}(\boldsymbol{x}|\zeta) = \sum_{j=1}^{J_{\beta,k}} w_{\beta,k}^{(j)} \mathcal{N}\left(\boldsymbol{x}; \boldsymbol{F}_{\beta,k-1}^{(j)}\zeta + \boldsymbol{d}_{\beta,k-1}^{(j)}, \boldsymbol{Q}_{\beta,k-1}^{(k)}\right) \quad (7)$$

where $J_{\gamma,k}$, $w_{\gamma,k}^{(i)}$, $\boldsymbol{m}_{\gamma,k}^{(i)}$, $\boldsymbol{P}_{\gamma,k}^{(i)}$ $(i = 1, 2, \ldots, J_{\gamma,k})$ are given model parameters that determine the shape of the birth intensity; $J_{\beta,k}$, $w_{\beta,k}^{(j)}$, $\boldsymbol{F}_{\beta,k-1}^{(j)}$, $\boldsymbol{d}_{\beta,k-1}^{(j)}$, $\boldsymbol{Q}_{\beta,k-1}^{(j)} (j = 1, 2, \ldots, J_{\beta,k})$ determine the shape of the spawning intensity of a target with previous state $\boldsymbol{\zeta}$.

Assume $\boldsymbol{m}$ and $\boldsymbol{P}$ denote the mean and covariance of the state variable, respectively. Based on the assumptions A1–A6, suppose the posterior intensity at time $k-1$ is a Gaussian mixture of the form

$$D_{k-1}(\boldsymbol{x}) = \sum_{i=1}^{J_{k-1}} w_{k-1}^{(i)} \mathcal{N}(\boldsymbol{x}; \boldsymbol{m}_{k-1}^{(i)}, \boldsymbol{P}_{k-1}^{(i)}) \quad (8)$$

the predicted intensity for time k can be written as

$$D_{k|k-1}(\boldsymbol{x}) = \sum_{i=1}^{J_{k|k-1}} w_{k|k-1}^{(i)} \mathcal{N}\left(\boldsymbol{x}; \boldsymbol{m}_{k|k-1}^{(i)}, \boldsymbol{P}_{k|k-1}^{(i)}\right) \quad (9)$$

then the posterior intensity at time k is a Gaussian mixture and is given by

$$\begin{aligned} D_k(\boldsymbol{x}) &= (1 - p_{\mathrm{D},k}) \sum_{i=1}^{J_{k|k-1}} w_{k|k-1}^{(i)} \mathcal{N}\left(\boldsymbol{x}; \boldsymbol{m}_{k|k-1}^{(i)}, \boldsymbol{P}_{k|k-1}^{(i)}\right) \\ &\quad + \sum_{z \in \boldsymbol{Z}_k} \sum_{j=1}^{J_{k|k-1}} w_{k|k-1}^{(i)} \mathcal{N}\left(\boldsymbol{x}; \boldsymbol{m}_k^{(i)}, \boldsymbol{P}_k^{(i)}\right) \end{aligned} \quad (10)$$

The posterior PHD is propagated via the PHD recursion by a calculation process similar to Kalman filter. Detailed process of the GM-PHD filter can be seen in Ref. [7].