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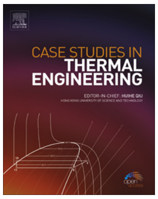

Case studies on the influence of microstructure voids on thermal conductivity in fractal porous media

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ABSTRACT

Several studies have shown that fractal geometry is a tool that can replicate and investigate the nature of the materials and their physical properties. The Sierpinski carpet is often utilized to simulate porous microstructures. By using this geometric figure it is possible to study the influence of pore size distribution on deterministic fractal porous media. The determination of the thermal conductivity can be carried out using the electrical analogy. So, microstructure models have been converted in electrical fractal patterns. This fractal procedure is characterized by a close relationship with the actual microstructure and prevent papers has been validate it with experimental data in a series of former papers. In this work it is possible to show how thermal conductivity changes in relation to pore size distribution and geometric microstructure parameters.

1. Introduction

Modeling is an important way to control a crucial aspect of our time: efficient energy management. A significant and often critical role is played by the availability of materials that will ensure an improvement in the performances when it comes to the insulation (or transmission) of energy systems.

Engineering and Materials Science are fields in constant search for innovations [1,2] and solutions that may gradually replace and improve former performances. It seems clear that the conductivity of materials is an important subject in order to classify them into different fields of application.

As a matter of fact porosity plays a paramount role and as a matter of fact, the presence of voids, appreciably affects the values of this physical magnitude. The effective thermal conductivity (k_{eff}) of porous materials is a function of the intrinsic characteristics of the solid phase (k_s) and of the fluid (k_f) one. The fluid phase occupies the pores (which may entail a number of solid phases and/or different fluid phases), from the fraction by volume of the voids and their size distribution [3,4]. The role of the volume fraction of vacuum has been widely investigated so far [5,6]. On the other hand the role of distribution has been studied much less [7,8].

This is due to the difficulty of controlling it in the course of the production process as well as to the difficulty of characterizing it from a geometrical point of view. In this respect the recent applications of concepts and methods related to Fractal Geometry are of great interest [9–11].

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Nomenclature

$\bar{c}$	conductance
D_f	fractal dimension
i	iteration
k_{eff}	effective thermal conductivity
k_f	fluid thermal conductivity
k_M	thermal conductivity for "material M"
k_s	solid thermal conductivity
l	length of the piece of material
n^{th}	step
r	ray
$\bar{R}$	resistence
$\bar{R}^1$	resistence for first line of pattern
$\bar{R}^2$	resistence for second line of pattern
$\bar{R}^3$	resistence for third line of pattern
$\bar{R}_s$	resistence of solid phase
$\bar{R}_f$	resistence of fluid phase
R_{eff}	total resistivity
R_f	fluid resistivity
R_s	solid resistivity

Greek symbol

ε	pore volume fraction

In this research the approach to the description of the porosity is fractal, while the physical-phenomenological section has been based on the analogy between the thermal conductivity and electrical conductivity. In a nutshell, the fractal microstructure will be considered as a fractal network of resistance arranged in series-parallel combinations.

2. Fractal model and fractal electrical approach

The proposed approach is characterized by a strict adherence to the pore size distribution of real materials. Its originality lies in being able to treat any type of real microstructures.

Fig. 1 shows different fractal models which were taken into account for this work and their pore size distribution, while Table 1 reports their input data. This procedure is based on: (a) determination of a fractal model based on the Sierpinsky carpet (4 type, with: 1, 2, 3 and 4 pores) with different pore size distribution with the aim of simulating experimental pore size distribution (maximum pore = 100 μm); (b) definition of the circuit diagram of the units, assuming that at each subsequent iteration of the scaling k_{eff} is the calculation result of the previous stage (Fig. 2).

As far as for the system in which there are 2 pores, by defining the smaller cell (Fig. 2) corresponding to the size of the smallest pores, $k_{eff}=f$ (circuit diagram corresponding to the microstructure) is provided. Being $\bar{c}$ and $\bar{R}$ respectively, conductance and average electrical resistance for a pattern in which pores are in different line (S2A, Fig. 1)

$$\bar{c}=\frac{1}{\bar{R}} \tag{1}$$

$$\bar{c}=\frac{1}{\bar{R}^1}+\frac{1}{\bar{R}^2}+\frac{1}{\bar{R}^3} \tag{2}$$

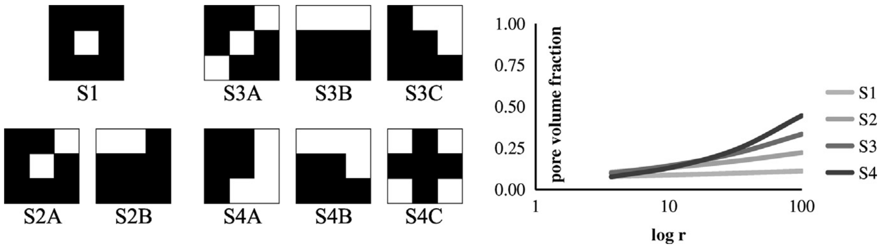

Fig. 1. Fractal models to study thermal conductivity and their pore size distributions.

Table 1
Input data for every Sierpinski unit base.

	D_f	Pore number	Iteration (i)	ε ($i_1/i_2/i_3/i_4$)
S1	1.89	1	4	0.11/0.21/0.29/0.37
S2	1.77	2	4	0.22/0.39/0.52/0.63
S3	1.63	3	4	0.33/0.55/0.70/0.80
S4	1.46	4	4	0.44/0.69/0.82/0.90