Chinese Journal of Aeronautics, (2015), **28**(3): 757–769

Chinese Society of Aeronautics and Astronautics
& Beihang University

Chinese Journal of Aeronautics

cja@buaa.edu.cn
www.sciencedirect.com

Aerodynamic optimization design for high pressure turbines based on the adjoint approach

Chen Lei [a,*], **Chen Jiang** [b]

[a] *Heavy Engineering Research Association, Auckland 2104, New Zealand*
[b] *School of Energy and Power Engineering, Beihang University, Beijing 100191, China*

Received 24 April 2014; revised 5 December 2014; accepted 2 March 2015
Available online 22 April 2015

KEYWORDS

Adjoint method;
Aerodynamic design;
High pressure turbine;
Optimization design;
Objective function

Abstract A first study on the continuous adjoint formulation for aerodynamic optimization design of high pressure turbines based on S_2 surface governed by the Euler equations with source terms is presented. The objective function is defined as an integral function along the boundaries, and the adjoint equations and the boundary conditions are derived by introducing the adjoint variable vectors. The gradient expression of the objective function then includes only the terms related to physical shape variations. The numerical solution of the adjoint equation is conducted by a finite-difference method with the Jameson spatial scheme employing the first and the third order dissipative fluxes. A gradient-based aerodynamic optimization system is established by integrating the blade stagger angles, the stacking lines and the passage perturbation parameterization with the quasi-Newton method of Broyden–Fletcher–Goldfarb–Shanno (BFGS). The application of the continuous adjoint method is validated through a single stage high pressure turbine optimization case. The adiabatic efficiency increases from 0.8875 to 0.8931, whilst the mass flow rate and the pressure ratio remain almost unchanged. The optimization design is shown to reduce the passage vortex loss as well as the mixing loss due to the cooling air injection.

1. Introduction

With the increasing need for high performance gas turbines to reduce the emission and the engine weight, the emerging trend is to use mathematical optimization techniques as an integral part of the aerodynamic design toolkit. Stochastic and gradient-based methods are the normal mathematical optimization algorithms. The stochastic method searches for a global optimal solution by monitoring the magnitude of the objective function, while the gradient-based method searches for a local optimal solution by monitoring the sensitivity of the objective function to the changes in the design variables. Both of the stochastic and the gradient-based methods tend to consume considerably computational resources for the cases with a large number of design variables.

The adjoint method is another gradient-based approach especially for the optimization with numerous design variables.

* Corresponding author. Tel.: +64 92624751.
E-mail addresses: lei.chen@hera.org.nz (L. Chen), chenjiang27@buaa.edu.cn (J. Chen).

Peer review under responsibility of Editorial Committee of CJA.

Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.cja.2015.04.022
1000-9361

In this method, the adjoint system is conducted in a similar way as that in the optimal control problems. By introducing the adjoint variable vectors, the gradients of the objective function with respect to the design variables are calculated indirectly by solving the adjoint equations. Therefore, the sensitivity analysis is almost independent of the numbers of the design variables, and solving two sets of flow equations in one design cycle is nearly the total computing cost. When the flow equations and the adjoint equations are fully converged, the final gradients of the objective function with respect to the design variables can be obtained efficiently.

Pironneau[1] was the first to use the adjoint method in fluid mechanics, and then Jameson[2] applied adjonit method in the aeronautical field. Combining the continuous adjoint method with CFD technology, Jameson developed the optimization design method which was applied to the transonic wing-body combinations.[3] Moreover, the discrete adjoint method was also developed.[4] "Continuous" and "discrete" methods symbolize two alternative approaches to deriving adjoint equations. As there is no clear quantitative comparison between the two approaches,[5,6] they seem to achieve the same optimal goals. Both of the methods have performed well in the optimization for airfoils,[7] wings,[8] wing-body configurations[9] and business jets.[10]

The adjoint method has been utilized in the area of turbomachinery. Li et al.[11,12] used the continuous adjoint method based on Navier–Stokes and Euler equation respectively to conduct the aerodynamic optimization design for turbine blades, and the optimization system was validated by several numerical cases. Papadimitriou and Giannakoglou[13] developed the continuous adjoint formulation to improve the aerodynamic performance of a 3D peripheral compressor blade cascade. Wang and He[14] first proposed the adjoint non-reflective mixing-plane treatment method, and carried out the aerodynamic blading shape optimization design in a multi-stage turbomachinery environment. Luo et al.[15] used the adjoint optimization method to reduce the secondary flow loss of turbine blades by redesigning the blade. Ji et al.[16] combined the continuous adjoint method with thin shear-layer Navier–Stokes equations to construct an efficient sensitivity analysis optimization system for multi-stage turbomachinery blades, and the adjoint optimization code was validated through two compressor blades design cases. Zhang and Feng[17] used the automatic differentiation tool to develop a discrete adjoint solver, and the optimization system was validated via a turbine cascade under the viscous flow environment.

For the aerodynamic design of gas turbines, S_2 surface design plays a crucial role in the entire design system.[18] Blade design is based on the simulation results of S_2 surface through flow calculation. This paper presents the results of the first study on the adjoint method applied to the S_2 surface through flow calculation including the cooling air effect. The adjoint method is combined with the Euler equation with the source term to develop an efficient sensitivity analysis model for turbine blades (stagger angles and stacking lines) and the passage in a specified objective function. The validation of the optimization system is carried out via the case of aerodynamic optimization of a high pressure turbine.

2. Flow equations and solution methods

The steady Euler equations of the curvilinear coordinates system are utilized to predict the aerodynamic performance of the gas turbine on S_2 surface. The effects of the viscous losses, the leakages and the cooling air on the flow are concerned as the source terms in the right part of the Eq. (1).

$$\frac{\partial}{\partial t}\left(\frac{r\boldsymbol{U}}{J}\right)+\frac{\partial}{\partial \xi}\widetilde{\boldsymbol{F}}+\frac{\partial}{\partial \eta}\widetilde{\boldsymbol{G}}=\widetilde{\boldsymbol{Q}} \tag{1}$$

where $\boldsymbol{U}$ is the conservative flow variable vector; $\widetilde{\boldsymbol{F}},\widetilde{\boldsymbol{G}}$ are the convective flux vectors in the curvilinear coordinate systems; $\widetilde{\boldsymbol{Q}}$ is the source term of the flow equations.

$$\boldsymbol{K}_{ij}=\left[\frac{\partial \xi_i}{\partial x_j}\right],\quad J=\det(\boldsymbol{K})$$

$$\widetilde{\boldsymbol{F}}=\frac{r}{J}\left(\boldsymbol{F}\xi_z+\boldsymbol{G}\xi_r+\boldsymbol{H}\frac{1}{r}\xi_\varphi\right)$$

$$\widetilde{\boldsymbol{G}}=\frac{r}{J}\left(\boldsymbol{F}\eta_z+\boldsymbol{G}\eta_r+\boldsymbol{H}\frac{1}{r}\eta_\varphi\right),\quad \widetilde{\boldsymbol{Q}}=\frac{\overline{\boldsymbol{h}}}{J}+\overline{\boldsymbol{h}_1}$$

$$\overline{\boldsymbol{h}}=\begin{bmatrix} r\dot{m} \\ r\dot{m}v_z+f_z \\ r\dot{m}v_r+f_r+\rho(w+\omega r)^2 \\ r\dot{m}v_\varphi+f_\varphi-\rho v(w+2\omega r) \\ r\dot{m}H'+\omega^2 r^2\rho v \end{bmatrix}$$

$$\overline{\boldsymbol{h}_1}=-\begin{bmatrix} 0 \\ \frac{r}{J}\zeta_z\left(\frac{\partial p}{\partial \zeta}\right)+p\frac{\partial}{\partial \zeta}\left(\frac{r}{J}\zeta_z\right) \\ \frac{r}{J}\zeta_r\left(\frac{\partial p}{\partial \zeta}\right)+p\frac{\partial}{\partial \zeta}\left(\frac{r}{J}\zeta_r\right) \\ \frac{\zeta_\varphi}{J}\left(\frac{\partial p}{\partial \zeta}\right)+p\frac{\partial}{\partial \zeta}\left(\frac{\zeta_\varphi}{J}\right) \\ 0 \end{bmatrix}$$

where terms $\xi_z,\xi_r,\xi_\varphi,\zeta_z,\zeta_r,\zeta_\varphi$ represent the partial derivatives $\frac{\partial\xi}{\partial z},\frac{\partial\xi}{\partial r},\frac{\partial\xi}{\partial\varphi},\frac{\partial\zeta}{\partial z},\frac{\partial\zeta}{\partial r},\frac{\partial\zeta}{\partial\varphi}$; $\boldsymbol{F}$, $\boldsymbol{G}$ and $\boldsymbol{H}$ are the convective flux vectors in the cylindrical coordinate system; $\dot{m}$ is the mass flow rate of the cooling air; v_z, v_r and v_φ are the velocity components of the cooling air, and the injection of the cooling air is classified into nine types as shown in Fig. 1; H' is the total enthalpy of cooling air; (f_z,f_r,f_φ) are for accounting of viscous losses effects,

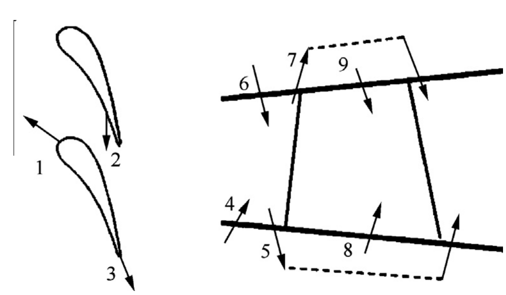

Fig. 1 Injection types of the cooling air.