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Acceptance sampling plan of accelerated life testing for lognormal distribution under time-censoring

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Abstract Lognormal distribution is commonly used in engineering. It is also a life distribution of important research values. For long-life products follow this distribution, it is necessary to apply accelerated testing techniques to product demonstration. This paper describes the development of accelerated life testing sampling plans (ALSPs) for lognormal distribution under time-censoring conditions. ALSPs take both producer and consumer risks into account, and they can be designed to work whether acceleration factor (AF) is known or unknown. When AF is known, life testing is assumed to be conducted under accelerated conditions with time-censoring. The producer and consumer risks are satisfied, and the size of test sample and the size of acceptance number are optimized. Then sensitivity analyses are conducted. When AF is unknown, two or more predetermined levels of accelerated stress are used. The sample sizes and sample proportion allocated to each stress level are optimized. The acceptance constant that satisfies producer and consumer risk is obtained by minimizing the generalized asymptotic variance of the test statistics. Finally, the properties of the two ALSPs (one for known-AF conditions and one for unknown-AF conditions) are investigated to show that the proposed method is correct and usable through numerical examples.

1. Introduction

Life testing sampling plans (LSPs) are usually used to determine whether to accept or reject batches of products when lifespan is an important index. There are many studies on the design of LSPs. The differences among them are mainly in the assumed lifetime distribution (exponential and Weibull lifetime distributions), censoring scheme (time-censoring or failure-censoring) and testing conditions (accelerated or use conditions). As science and technology have improved, product reliability has increased and product life has been extended. This makes it difficult for traditional reliability and life demonstration testing to judge product indexes. For example, in the traditional reliability demonstration testing scheme under exponential distribution, when the producer risk and consumer risk are 20%, the testing time is 4.3 times longer than the MTBF inspection limit. That means if the MTBF is 2000 h, the testing time is 8600 h. This is unacceptably long.

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Accelerated life testing (ALT) is used to facilitate fast acceptance. This shortens testing time and reduces development costs through the use of harsher-than-use testing conditions.

There are many studies on the design of LSPs that involve exponential lifetime distribution. In 1980, Spurrier and Wei[1] developed Type-I censoring and produced LSPs that consider only the producer's risk. In 1995, Jeong and Yum[2] expanded this design to cases in which both types of risk are considered. Kim and Tum[3] presented another LSPs design involving Type-I censoring and intermittent monitoring in 2010. Muhammad Aslam et al.[4] published a study on acceptance sampling plans for generalized exponential distribution when the lifetime experiment is terminated at a pre-determined time. For Weibull distribution, Fertig and Mann[5] discussed sampling plans for use with Weibull distribution and constructed the hybrid censored LSPs. In 2000, Balasooriya et al.[6] first presented progressively failure-censored LSPs. In 2004, Balasooriya and Low[7] expanded upon the results of the previous Balasooriya[6] study to cases involving competing causes of failure. In 2004, Chen et al.[8] presented Bayesian LSPs under hybrid censoring conditions with prior information on the shape and scale parameters of the Weibull lifetime distribution. In 2009, Aslam and Jun[9] produced a group acceptance LSP for a truncated life testing. This LSP was able to test multiple items simultaneously. In 2013, Ismail[10] designed a step-stress accelerated life test under failure-censoring conditions assuming the Weibull distribution with Type-II censored data.

In engineering, lognormal distribution plays an important role in statistically predicting the fatigue life of mechanical products. In 1962, Gupta[11] studied life testing sampling plans for truncated life tests from the normal and lognormal distributions. In 1989, Schneider[12] discussed the design of variable-sampling plans based on failure-censored samples. This method of design can be applied to lognormal and Weibull-distributed lifetimes. In 2000, Balasooriya and Balakrishnan[13] presented sampling plans for lognormal distribution. These plans are based on progressively censored samples. Large-sample approximations of the best linear unbiased estimators of the location and scale parameters are used. In 2006, Wu and Lu[14] proposed a statistical method for working out reliability sampling plans with Type-I censored samples for items whose failure times have either normal or lognormal distributions. In 2009, Srivastava and Shukla[15] presented an optimum simple ramp accelerated life test with two different linearly increasing stresses for log–logistic distribution under Type-I censoring.

In accelerated testing, when the acceleration factor (AF) is unknown, for exponential distribution, Yum and Kim[16] developed a life sampling plan for failure-censoring at two stress levels. However, the calculations required for this plan are very complicated and the error rate is large. In 1994, Hsieh[17] expanded upon Yum and Kim's work such to minimize the total number of failures at the stress level. In 1993, Bai et al.[18] became the first to develop the LSP for use under failure-censoring at two stress levels above ordinary-use conditions. Then, in order to study time censoring test plans, they[19] extended the case under expected test time constraints. In 2009, Seo et al.[20] designed accelerated life testing sampling plans (ALSPs) for cases in which the shape parameter of Weibull distribution is non-constant. Kim and Yum[21] designed an ALSP under time-censoring conditions by assuming that the shape parameter of Weibull distribution was unknown. Then they developed ALSPs under hybrid censoring conditions by assuming that the shape parameter of Weibull distribution was known.[22]

In engineering practice, time censoring is the most common way to cut a test short. For long-life products that follow lognormal distribution, it is necessary to develop ALSPS under time-censoring conditions. In this way, this paper discusses the design of ALSPs for lognormal distribution under time-censoring conditions. Taking both producer and consumer risks into consideration. ALSPs can be designed whether AF is known or not.

In Section 2, we depict the design of an accelerated life testing sampling plan based on the lognormal distribution under time-censoring conditions when AF is known and when it is unknown. In Section 3, we apply these two sampling plans to case study and sensitivity analysis is also conducted. Finally, conclusions are presented in Section 4.

2. Accelerated life testing sampling plan under time-censoring

2.1. Assumptions

Assumption 1. The lifetime of products under any stress levels follows lognormal distribution, and the cumulative distribution function can be expressed as

$$F(t) = \int_0^t \frac{1}{\sqrt{2\pi}\sigma x} \mathrm{e}^{-\frac{1}{2}\left(\frac{\ln x-\mu}{\sigma}\right)^2} \mathrm{d}x = \Phi\left(\frac{\ln t-\mu}{\sigma}\right) \tag{1}$$

where μ is the location parameter and σ the scale parameter.

Assumption 2. The location parameter μ is acceleration model and satisfies a linear function of stress s. It can be written as follows:

$$\mu = \gamma_0 + \gamma_1 \varphi(s) \tag{2}$$

where γ_0 and γ_1 are the unknown parameters. Function $\varphi(s)$ is the function of stress s and could be different representations when different accelerate stresses are used. For example, if temperature is chosen, $\varphi(s) = 1/s$; if electrical stress, $\varphi(s) = \ln s$ or $\varphi(s) = s$. In addition, the acceleration factor is $\mathrm{AF} = \mu_\mathrm{U}/\mu_\mathrm{A}$. Here, subscript "U" represents "use conditions" and "A" stands for "accelerated conditions." μ_U and μ_A are the mean of log lifetime of lognormal distribution under use and accelerated conditions, respectively.

Assumption 3. Failure mechanism will not change with stress levels, that means, the scale parameter σ keeps constant at different stress levels.

2.2. Accelerated life testing sampling plan when AF is known

The key to using the accelerated testing technology in LSPs is dealing with the acceleration factor. This section first discusses how to design ALSP when AF is known.

Since AF has been confirmed in advance, the life of product under accelerated conditions can be directly determined based on use conditions. Then the reliability of product at the time of censoring could be determined under accelerated conditions by using the operating characteristic (OC) curve. We can solve the equation regarding sample size n and acceptance number c