Chinese Journal of Aeronautics, (2015),28(2): 403–409

Chinese Society of Aeronautics and Astronautics
& Beihang University

Chinese Journal of Aeronautics

cja@buaa.edu.cn
www.sciencedirect.com

Atomization mechanism of a charged viscoelastic liquid sheet

Liu Lujia [a], Liu Yingjie [b,*], Lu Lipeng [b]

[a] *School of Astronautics, Beihang University, Beijing 100191, China*
[b] *School of Energy and Power Engineering, Beihang University, Beijing 100191, China*

Received 7 October 2014; revised 6 November 2014; accepted 21 November 2014
Available online 21 February 2015

KEYWORDS

Absolute growth rate;
Absolute instability;
Electrified sheet;
Spatial–temporal instability;
Viscoelastic liquid

Abstract In order to study atomization mechanism of a viscoelastic liquid sheet in an electric field, the spatial–temporal stability analysis of a viscoelastic liquid sheet injected into a dielectric stationary ambient gas in the presence of a vertical electric field is conducted. The dispersion relations of both sinuous and varicose disturbance modes are solved to explore the spatial–temporal instability of a charged viscoelastic sheet, by setting both the wave number and frequency complex. A parametric study is performed to test the influence of the dimensionless parameters on the absolute instability of the sheet. The results show that the increase of liquid Weber number and time constant ratio, or decrease of gas to liquid density ratio and Reynolds number, can damp the absolute instability. The effect of the liquid elasticity depends on the value of time constant ratio: when time constant ratio is small, the increase of liquid elasticity could amplify absolute growth rate, but the effect is weak when the elasticity number is relatively large; when time constant ratio is large, the increase of liquid elasticity cannot affect the absolute growth rate. Moreover, the variation of electrical Euler number can hardly influence the absolute instability of a charged viscoelastic sheet.

1. Introduction

In recent years, with advantages of short development cycle, low cost, high reliability and being able to complete work that many large, complicated and expensive satellites cannot complete when used for formation flying and constellation positioning, the tiny spacecraft has made itself concerned by all the aspects both at home and abroad, and has obtained rapid development and application. In view of the developing trend of current space propulsion, electrospray micro propulsion system is considered as one of the best choices of small satellite propulsion control systems. In this case, as one kind of electrospray micro propulsion, colloid propulsion should be studied seriously. In other words, studies on atomization mechanism of a viscoelastic liquid sheet in an electric field are of great significance.

Numerous efforts have been devoted to gaining insight into the behavior of liquids under the influence of aerodynamic and capillary forces and subjected to electric fields. It is well-known

* Corresponding author. Tel.: +86 10 82339571.
E-mail addresses: iammyself0101@126.com (L. Liu), liuyingjie@buaa.edu.cn (Y. Liu), lulp@buaa.edu.cn (L. Lu).
Peer review under responsibility of Editorial Committee of CJA.

ELSEVIER Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.cja.2014.12.034
1000-9361 © 2015 Production and hosting by Elsevier Ltd. on behalf of CSAA & BUAA.

that a circular jet and a thin planar sheet are two basic forms in which a liquid issues from an injector. The stability of circular liquid jets subjected to electric fields has been studied extensively. Basset[1] first studied the stability of perfectly conducting jets subjected to axisymmetric disturbances. Huebner and Chu[2] investigated the stability of charged inviscid jets subjected to both axisymmetric and non-axisymmetric disturbances. Saville[3] examined the stability of perfectly conducting Newtonian jets of arbitrary viscosity stressed in a radial electric field. Baudry et al.[4] and Artana et al.[5] checked the effects of electrical field, jet velocity, liquid properties and ambient gas on the temporal growth rate of inviscid liquid jets in the radial electrical field. Mestel[6] examined the stability of viscous liquid jets in the axial electrical field. Li et al.[7] and Yang et al.[8] explored the effect of liquid elasticity in the process of electrospinning by studying the stability of electrified viscoelastic liquid jets.

Comparatively, the stability of planar liquid sheets subjected to electric fields is less studied. Traveling waves of arbitrary amplitudes and wavelengths on liquid sheets when an electric field acts has been studied by Papageorgiou and Vanden-Broeck.[9,10] In their studies, the fluid is taken to be inviscid, incompressible and non-conducting. El-Sayed[11] studied the electro-aerodynamic instability of a dielectric, compressible liquid sheet streaming into a stationary compressible gas in the presence of a uniform horizontal electric field. The results show that the electric field is found to have a stabilizing effect and there exists a critical Weber number above which instability is suppressed by the surface tension effect. Yang et al.[12] investigated the instability characteristics of the electrified viscoelastic liquid sheets. The linear viscoelastic constitutive relation is used to describe the viscoelasticity of dilute polymer solutions under small or moderate deformation. Besides, a number of researches have devoted their efforts to describe the instability of viscoelastic jets or sheets in the absence of electric environments.[13–18]

In all the above-mentioned references, however, the studies are confined to the scope of temporal stability analysis. They analyzed the temporal stability of this base state by imposing, at $t = 0$, an interfacial disturbance with Fourier wave numbers k in the x direction. They determined the evolution of this initial disturbance using normal modes $\eta = \hat{\eta}\exp(ikx + \omega t)$, where x is along the thread axis and ω is complex. They used normal mode analysis to linearize the governing equations, constitutive equations and boundary conditions, and solve for the complex angular frequency ω in terms of the real wave number k. The temporal theory only provides information about the growth rate and whether a system is temporally stable or unstable. The theory is incapable of describing the additional spatial–temporal behavior observed in experiments, which indicates that the wave starts after the liquid is ejected from a tube and develops downstream along the sheet.[19]

A more physically consistent analysis can be made in terms of spatial–temporal instability by treating both k and ω complex.[20–21] There are two kinds of instabilities in spatial–temporal evolving disturbances: the disturbances can only be convected downstream in convective instability, but in absolutely instability the disturbances are propagated in the downstream, as well as in the upstream direction.[22] This concept was first developed in the context of plasma physics[23] and later on applied to optics and hydrodynamics.[24] Monkewitz[25] showed that the sequence of transitions behind a cylinder wake takes place as follows when the Reynolds number is increased: transition from stability to convective instability, transition from convective to local absolute instability and finally transition to a self-sustained global mode when a sufficiently large portion of the flow has become absolutely unstable. It is of considerable interest to know if a similar transition between absolute instability and convective instability would take place for a charged viscoelastic liquid sheet.

In this paper, the atomization mechanism of a charged viscoelastic liquid sheet is reported by the method of spatial–temporal instability analysis. The effects of flow parameters on the spatial–temporal stability of a charged viscoelastic sheet are examined by observing whether an increase of the value of parameter tends to increase or decrease the value of absolute growth rate.

2. Mathematical formulation of the problem

A two-dimensional viscoelastic incompressible liquid sheet of thickness $2a$, moving through a quiescent, inviscid, incompressible gas medium, is examined here. The coordinates are chosen so that the x-axis is parallel to the direction of the liquid sheet flow and the y-axis is normal to the liquid sheet with its origin located at the middle plane of the liquid sheet. The basic velocity of the sheet is steady and uniform, denoted by $\boldsymbol{U} = (U, V)$. The sheet has only a nonzero axial component in the present study, i.e., V is assumed to be zero. The liquid and gas have densities of ρ_l and ρ_g respectively, and the surface tension of liquid is σ. To electrify the liquid sheet, two flat electrodes are set on the top and bottom surfaces of the liquid sheet respectively and the distance between the flat electrode and the liquid sheet is d. A voltage V_0 is imposed between the sheet surface and electrodes. The liquid is perfect conductor and the ambient gas is perfect dielectric. A basic electric field of magnitude $-V_0/d$, which is vertical to the sheet surface, is thus formed in the ambient gas. The density of free charge on the unperturbed gas–liquid interface is $-\varepsilon_0 V_0/d$, where ε_0 is dielectric constant of ambient gas.

If a protuberance is produced on the interface owing to any disturbance, forces acting on the interfaces develop. The surface tension always tends to restore the interface back into its original equilibrium position, while the disturbance generally enhances the degree of instability, i.e., increase the amplitude of the disturbance. A relative velocity between the liquid and gas promotes the growth of disturbances until the liquid sheet disintegrates into fragments. In this paper there are two types of disturbances for liquid sheet. When the distortions on two liquid–gas interfaces are in-phase, it is called sinuous disturbance wave (see Fig. 1(a)); while the phase contrast of distortions on two interfaces is 180° and it is called varicose disturbance wave (see Fig. 1(b)).

The linear stability analysis of electrified liquid sheets is different from that of non-electrified sheets because the effect of electrical Maxwell tensor should be taken into account. The derivation of the dispersion relation is standard and we just simply outline the derivation procedure of the dispersion relation for sinuous disturbances. The detailed derivation of dispersion relation of an electrified viscoelastic liquid sheet refers to Yang et al.[12]

The governing equations of liquid are the conservation laws of mass and momentum: