Chinese Journal of Aeronautics, (2015),28(2): 457–468

Chinese Society of Aeronautics and Astronautics
& Beihang University
Chinese Journal of Aeronautics
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Response regimes of narrow-band stochastic excited linear oscillator coupled to nonlinear energy sink

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Received 13 June 2014; revised 11 August 2014; accepted 22 December 2014
Available online 23 February 2015

KEYWORDS

Nonlinear energy sink;
Path integration method;
Stochastic excitation;
Targeted energy transfer;
Vibration absorption

Abstract This paper draws attention to the issue of the vibration absorption of nonlinear mechanical system coupled to nonlinear energy sink (NES) under the impact of the narrow band stochastic excitation. Firstly, based on the complex-averaging method and frequency detuning methodology, response regimes of oscillators have been researched under the linear impact of coupling a nonlinear attachment with less relativistic mass and an external sinusoidal forcing, of which results turn out that the quasi-periodicity response regime of system which occurs when the external excitation amplitude exceeds the critical values will be the precondition of the targeted energy transfer. Secondly, basing on the path integration method, vibration suppression of NES has been researched when it is affected by a main oscillator with a narrow band stochastic force in the form of trigonometric functions, of which results show that response regimes are affected by the amplitude of stochastic excitation and the disturbance strength. Finally, all these conclusions have been approved by the numerical verification and coincided with the theoretical analysis; meanwhile, after the comparing analysis with the optimal linear absorber, it turns out that the NES which is affected by the narrow band stochastic force could also suppress the vibration of system with a better effect.

1. Introduction

The research of adding a strong nonlinear light-weight attachment on the linear oscillator has attracted a lot of attention. A significant number of researches show that the system which uses the targeted energy transfer (TET) as mechanism could offer higher energy transfer efficiency.[1–5] Meanwhile, the energy exchange phenomenon in the non-conservative systems could be understood or explained as the nonlinear normal modes (NNMs).[1] There is huge energy exchange existing between different NNMs. Under a certain condition, the localization will occur in the energy pumping.[6,7] Using this localization and irreversible transfer, the nonlinear energy sink (NES) realizes its high efficiency vibration absorber and vibration suppression.[8–10]

It is Gendelman et al. who firstly analyze the dynamic behavior of ideal Hamilion system and supply the numerical proof of the TET from the linear oscillator to the nonlinear

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Peer review under responsibility of Editorial Committee of CJA.

Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.cja.2015.02.010
1000-9361

oscillator, which means that, under some certain condition, the energy sink could be used in the vibration absorption.[6,8,11] To obtain a more essential understanding of the TET, Manevitch et al. proposed a new method, on the basis of the complex variable substitution, which could be used to research the energy pumping in the non-conservative system when the initial excitation is the impact loading.[12–14] During the strongly nonlinear oscillation, the NNMs have been applied widely, especially in the research of NES. The paper processes theoretical analysis and experimental research on coupling NES linear oscillator, of which results show that the system could present two response regimes with different essence, one is steady-state regime, and the other is the quasi-periodic regime which could supply the high-efficient vibration suppression ability.[15–18] Meanwhile, the numerical simulation illustrates that the quasi-periodic regime also could happen under the narrow band stochastic impact. However, there is barely research on the effect of vibration suppression of NES under this impact, but there is a large amount of literature researching NES's application.[19–23]

In Refs. 24,25, it was the detuning parameter theory which proved that the performance of NES depends on the amplitude of the external sine force and NES could have higher absorption efficiency in a wide range of forcing amplitudes. In Ref. 26, the same method verified that there exists the quasi-periodic regime in the system under a narrow band stochastic force in the form of trigonometric functions. However, without strong evidence, vibration suppression of NES is still unknown.

One of the main research aims of this paper is the linear oscillator coupled to an NES, the dynamic behavior of system and the response regimes under the narrow band stochastic excitation. It was easy to find out in the relevant literature that the quasi-periodic regime has the vibration suppression ability. Meanwhile, because the mathematical modes of the narrow band stochastic excitation are the same as the sine excitation, this paper firstly researches the response of oscillator affected by the sine excitation and discusses the suppression effect of the NES by the path integration method (PI).[27,28]

The structure has been arranged as follows: Section 2 contains two subsections, and one analyzes the dynamic behavior of system under the sine force using the detuning parameter theory, and the other analyzes the dynamic feature of system under the narrow band stochastic excitation; Section 3 focuses on some numerical simulation and verification; Section 4 summarizes the main achievements of the contemporary research.

2. Mathematical models of system under consideration

The system under consideration is depicted in Fig. 1. It consists of a linear oscillator coupled to a nonlinear attachment

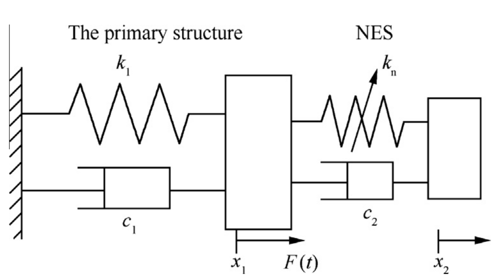

Fig. 1 Schematic of linear oscillator coupled to NES.

(pure cubic nonlinearity and linear damp). The primary structure (linear oscillator) is subjected to various different external disturbances such as impact loading, periodic or random forcing, and we only focus on the narrow-band stochastic excitation in this paper.

The dynamic motion of this system is given by the set of equations

$$\begin{cases} m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 + c_2(\dot{x}_1 - \dot{x}_2) + k_2(x_1 - x_2)^3 = F(t) \\ m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1)^3 = 0 \end{cases} \tag{1}$$

After change of variables, the system can be described by the following equations:

$$\begin{cases} \ddot{x}_1 + \varepsilon\lambda_1\dot{x}_1 + \omega_0^2x_1 + \varepsilon\lambda_2(\dot{x}_1 - \dot{x}_2) + k_n(x_1 - x_2)^3 = f(t) \\ \varepsilon\ddot{x}_2 + \varepsilon\lambda_2(\dot{x}_2 - \dot{x}_1) + k_n(x_2 - x_1)^3 = 0 \end{cases} \tag{2}$$

where m_1 and m_2 are the mass of primary oscillator and NES, k_1 and k_2 are the linear stiffness and nonlinear stiffness, c_1 and c_2 represent the damp of primary oscillator and NES, x_1 and x_2 refer to the displacement of two oscillators respectively, $\varepsilon \ll 1$ is a small parameter which represents the mass ratio of the linear oscillator and the attachment, $\lambda_1 = c_1/m_2$ and $\lambda_2 = c_2/m_2$ relate to damping coefficient, $k_n = k_2/m_1$ is the cubic stiffness and $\omega_0^2 = k_1/m_1$ is natural frequency of the primary structure and $f(t) = F(t)/m_1$ denotes the external force. In Eq. (2), all coefficients are adopted to be of order unity for the simplicity of the analytical treatment.

A change of state variables:

$$\begin{cases} u_1 = x_1 + \varepsilon x_2 \\ u_2 = x_1 - x_2 \end{cases} \tag{3}$$

By applying several simple algebraic simplification, rewriting Eq. (2) yields

$$\begin{cases} \ddot{u}_1 + \omega_0^2u_1 + \dfrac{\varepsilon}{1+\varepsilon}\lambda_1\dot{u}_1 + \dfrac{\varepsilon}{1+\varepsilon}\omega_0^2(u_2 - u_1) = f(t) \\ \ddot{u}_2 + \omega_0^2u_2 + \dfrac{\varepsilon}{1+\varepsilon}\lambda_1\dot{u}_1 + \dfrac{1}{1+\varepsilon}\omega_0^2(u_1 - u_2) \\ \quad +(1+\varepsilon)\lambda_2\dot{u}_2 + \dfrac{1+\varepsilon}{\varepsilon}k_nu_2^3 = f(t) \end{cases} \tag{4}$$

2.1. Analytical study for harmonic force with a tuning method

The form of narrow-band excitation is defined as Wedig,[29] which is expressed by

$$f(t) = \varepsilon A\cos(\Omega t + \sigma W(t)) \tag{5}$$

where εA is the amplitude of stochastic excitation, Ω the center frequency, $W(t)$ standard Wiener excitation. σ is a tuned parameter and represents the disturbance strength and bandwidth of the random excitation, and σ is a rather small value for narrow-band excitation. We analyze the response regimes of system under harmonic force with a tuning method, because the form of harmonic force is similar to random excitation.

Response regimes of system with NES under periodic forcing are studied in a large amount of literature.[7,17,18] Therefore, we briefly discuss the performance of vibration mitigation of a linear oscillator coupled to a single degree of freedom (DOF)