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Singular formalism and admissible control of spacecraft with rotating flexible solar array

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Abstract This paper is concerned with the attitude control of a three-axis-stabilized spacecraft which consists of a central rigid body and a flexible sun-tracking solar array driven by a solar array drive assembly. Based on the linearization of the dynamics of the spacecraft and the modal identities about the flexible and rigid coupling matrices, the spacecraft attitude dynamics is reduced to a formally singular system with periodically varying parameters, which is quite different from a spacecraft with fixed appendages. In the framework of the singular control theory, the regularity and impulse-freeness of the singular system is analyzed and then admissible attitude controllers are designed by Lyapunov's method. To improve the robustness against system uncertainties, an H_∞ optimal control is designed by optimizing the H_∞ norm of the system transfer function matrix. Comparative numerical experiments are performed to verify the theoretical results.

1. Introduction

In order to maximize the efficiency of solar electric power generation, spacecraft nowadays are usually equipped with solar array drive assemblies (SADAs) to keep their solar arrays continuously facing the Sun. However, system dynamics and control of the spacecraft with movable flexible appendages become more complex than those with fixed appendages. Without loss of generality, the attitude control of a slightly complex spacecraft which consists of a central rigid body and a flexible solar array rotating slowly with an orbital angular velocity along the pitch axis, will be considered in this paper. Numerous spacecraft can be treated as this sort of system, such as the satellite examples in the Engineering Test Satellite (ETS) series of Japan, the Geostationary Operational Environmental Satellite (GOES) series of US, the Indian Remote-Sensing (IRS) series of India, and the Satellite Pour l'Observation de la Terre (SPOT) series of France. Due to the rotation of the solar array, these spacecraft are typical linear parameter varying models in which the system modal frequencies, the inertia matrix of the spacecraft, and the coupling matrices between the spacecraft attitude motions and solar array vibrations and rotation vary continuously. Generally speaking, none of these problems will arise in spacecraft with fixed appendages. One of the feasible solutions to this kind of control problem is to choose a particular position of the solar array, e.g., 45°, as the worst-case configuration and design robust controllers, such as the loop-shaping H_∞ controller for satellite SPOT-4.[1] This

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controller was synthesized using the H_∞ coprime factorization approach and its robustness against parameter uncertainties was checked by mixed real/complex μ analysis. Using a cone complementarity linearization algorithm, low-order H_∞ controllers were developed such that the control scheme may be more attractive for practical use.[2] An alternative solution is to decompose this time-varying three-axis-coupled multi-input multi-output control system into three decoupled time-invariant single-input single-output subsystems, in which dynamic equations are transformed into solar array-fixed reference frames from body-fixed frames,[3–5] and then design robust controllers such as linear-quadratic-Gaussian (LQG) and H_∞. This method makes use of the property that the reference frame of the sun-tracking solar array is actually an inertial frame. However, as was pointed out in Ref.[6], this decoupling technique is an approximation which requires that the inertia around the roll/yaw axes of the central rigid body be symmetrical. With the development of onboard computational capabilities, various sophisticated control algorithms in the framework of gain scheduling are studied. However, scheduling algorithms of control gains may be time and memory consuming. Thus Ref.[7] developed a linear interpolation law of the gains such that the gain-scheduling algorithm can be easily implemented in an onboard satellite computer. As reported in Ref.[8], the gain scheduling control scheme has been tested by ETS-VIII attitude control experiments on-orbit and compared with other control approaches such as the classical PID control. However, generally almost all of the control strategies mentioned above neglected the rotation dynamics of the solar array.

Different from the above work, this paper will reduce the spacecraft model to a singular system by taking into account the dynamics of the rotating flexible solar array. Letting $\boldsymbol{x} \in \mathbf{R}^n$ ($n \geqslant 2$) denote a state vector and $\boldsymbol{f}: \mathbf{R}^n \times \mathbf{R}^n \times \mathbf{R}_+ \to \mathbf{R}^n$ denote a vector function, the mathematical model of the so-called singular systems is of the following form:

$$\boldsymbol{M}(t)\ddot{\boldsymbol{x}}(t) = \boldsymbol{f}(\dot{\boldsymbol{x}}, \boldsymbol{x}, t) \tag{1}$$

where the coefficient matrix $\boldsymbol{M}(t)$ of the acceleration term is denominated as the generalized mass matrix, and $\det \boldsymbol{M}(t) \equiv 0$. Singular systems are also called differential-algebraic equation systems since some algebraic constraints can be found explicitly in Eq. (1) besides differential equations. Thus some of the state variables in singular systems are redundant and should in principle be removed. However, when redundant variables are of practical interest and importance (like the rotation and flexible vibrations of the appendage in the spacecraft system under consideration), they should not be simply canceled. Moreover, redundant variables could not be derived explicitly if algebraic constraints are transcendental equations. On the other hand, some physical phenomena such as impulse and non-causality can be modeled and treated properly in the framework of a singular system instead of the classical state-space system.[9] As a natural and convenient modeling technique and a generalized control method, the singular system theory has been applied to large scale systems, economic processes, electrical networks, and complex mechanical systems, etc.

This paper is organized as follows. In Section 2, the dynamics and kinematics of the spacecraft are discussed in detail. Based on the modal identities, mathematical singularity of the spacecraft model is investigated and a generalized state-space formulation is presented in Section 3. In Section 4, an analysis in terms of the regularity and impulse-free property of the system is performed in the framework of singular control theory. On this basis, admissible attitude control laws with output feedback are constructed by Lyapunov's method. Then an H_∞ optimal control is introduced to improve the system robustness against the model and variable uncertainties. Numerical simulations are carried out based on the finite element analysis data of GOES-8 satellite in Section 5 to verify the controller designs and make a comparison with other controllers. Concluding remarks are made and some possible research topics are proposed in Section 6.

2. Spacecraft attitude dynamics and problem statement

Complex spacecraft usually have several large, flexible, and/or movable appendages connected to their core bodies. An accurate kinetic model of these spacecraft will contain quite a number of high-order nonlinear terms, and its system analysis and control design may become very difficult if not impossible. For tractability several assumptions are necessary to simplify the complex spacecraft model in practice, namely: (i) the orbital and the attitude motions of the spacecraft are decoupled from each other (i.e., orbital dynamics will not be considered in attitude dynamics); (ii) the center of mass of the spacecraft will not change in spite of the rotations and vibrations of the solar array; and (iii) variables of the attitude motion of the spacecraft, and those of the rotations and vibrations of the solar array are all first-order infinitesimal variables such that their products are high-order infinitesimal terms and can be neglected in the mathematical model.

Let $\boldsymbol{\omega} = [\omega_x \ \omega_y \ \omega_z]^{\mathrm{T}} \in \mathbf{R}^3$, $\boldsymbol{\Omega} \in \mathbf{R}$, and $\boldsymbol{\eta} = [\eta_1 \ \eta_2 \ \ldots \ \eta_N]^{\mathrm{T}} \in \mathbf{R}^N$ denote respectively the angular velocities of the spacecraft, the angular velocity of the solar array, and modal coordinates with truncation number N, one of the well-known formulations of the attitude dynamics of the spacecraft system, as shown in Fig. 1, is then presented as follows[10,11]

$$\begin{cases} \boldsymbol{J}\dot{\boldsymbol{\omega}} + \boldsymbol{J}_{\mathrm{bs}}\boldsymbol{l}\dot{\Omega} + \boldsymbol{H}_{\mathrm{bs}}\ddot{\boldsymbol{\eta}} = \boldsymbol{T} + \mathrm{HOT}_1(\boldsymbol{\omega}, \Omega, \dot{\boldsymbol{\eta}}, \boldsymbol{\eta}) \\ \boldsymbol{l}^{\mathrm{T}}\boldsymbol{J}_{\mathrm{bs}}^{\mathrm{T}}\dot{\boldsymbol{\omega}} + \boldsymbol{l}^{\mathrm{T}}\boldsymbol{I}_{\mathrm{s}}\boldsymbol{l}\dot{\Omega} + \boldsymbol{l}^{\mathrm{T}}\boldsymbol{H}_{\mathrm{s}}\ddot{\boldsymbol{\eta}} = \tau + \mathrm{HOT}_2(\boldsymbol{\omega}, \Omega, \dot{\boldsymbol{\eta}}, \boldsymbol{\eta}) \\ \boldsymbol{H}_{\mathrm{bs}}^{\mathrm{T}}\dot{\boldsymbol{\omega}} + \boldsymbol{H}_{\mathrm{s}}^{\mathrm{T}}\boldsymbol{l}\dot{\Omega} + \ddot{\boldsymbol{\eta}} = -\boldsymbol{D}\dot{\boldsymbol{\eta}} - \boldsymbol{K}\boldsymbol{\eta} + \mathrm{HOT}_3(\boldsymbol{\omega}, \Omega, \dot{\boldsymbol{\eta}}, \boldsymbol{\eta}) \end{cases} \tag{2}$$

where $\mathrm{HOT}_1(\boldsymbol{\omega}, \boldsymbol{\Omega}, \dot{\boldsymbol{\eta}}, \boldsymbol{\eta})$, $\mathrm{HOT}_2(\boldsymbol{\omega}, \boldsymbol{\Omega}, \dot{\boldsymbol{\eta}}, \boldsymbol{\eta})$ and $\mathrm{HOT}_3(\boldsymbol{\omega}, \boldsymbol{\Omega}, \dot{\boldsymbol{\eta}}, \boldsymbol{\eta})$ are the high-order infinitesimal terms that do not contain any acceleration variables ($\dot{\boldsymbol{\omega}}$, $\dot{\boldsymbol{\Omega}}$, or $\ddot{\boldsymbol{\eta}}$). $\boldsymbol{l} = [0 \ 1 \ 0]^{\mathrm{T}}$ is the rotational direction of the solar array, which means the solar array is rotating along the pitch axis of the spacecraft. The $N \times N$ diagonal matrices $\boldsymbol{D}$ and $\boldsymbol{K}$ are the orthonormal modal damping and stiffness of the flexible appendage, and both are strictly positive definite. Note that the inertia matrix of the whole spacecraft with respect to (w.r.t.) the central body-fixed reference frame F_{b} is

$$\boldsymbol{J} = \boldsymbol{J}_{\mathrm{b}} + \boldsymbol{J}_{\mathrm{s}} \tag{3}$$

where $\boldsymbol{J}_{\mathrm{b}}$ is the inertia matrix of the central body, and $\boldsymbol{J}_{\mathrm{s}}$ is that of the solar array, which is expressed explicitly by[10]

$$\boldsymbol{J}_{\mathrm{s}} = \boldsymbol{C}_{\mathrm{bs}}\boldsymbol{I}_{\mathrm{s}}\boldsymbol{C}_{\mathrm{bs}}^{\mathrm{T}} + m_{\mathrm{s}}\tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}^{\mathrm{T}} + \tilde{\boldsymbol{b}}\boldsymbol{C}_{\mathrm{bs}}\tilde{\boldsymbol{c}}_{\mathrm{s}}^{\mathrm{T}}\boldsymbol{C}_{\mathrm{bs}}^{\mathrm{T}} + \boldsymbol{C}_{\mathrm{bs}}\tilde{\boldsymbol{c}}_{\mathrm{s}}\boldsymbol{C}_{\mathrm{bs}}^{\mathrm{T}}\tilde{\boldsymbol{b}}^{\mathrm{T}} \tag{4}$$

where m_{s} is the total mass of the solar array and $\boldsymbol{I}_{\mathrm{s}}$ is the inertia matrix of the solar array w.r.t. the solar array-fixed reference frame F_{s}. $\tilde{\boldsymbol{b}}$ and $\tilde{\boldsymbol{c}}_{\mathrm{s}}$ are respectively the skew-symmetric matrices