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Description of shape characteristics through Fourier and wavelet analysis

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Abstract In this paper, Fourier and Wavelet transformation were adopted to analyze shape characteristics, with twelve simple shapes and two types of second phases from real microstructure morphology. According to the results of Fast Fourier transformation (FFT), the Fourier descriptors can be used to characterize the shape from the aspects of the first eight Normalization amplitudes, the number of the largest amplitudes to inverse reconstruction, similarity of shapes and profile roughness. And the Diepenbroek Roughness was rewritten by Normalization amplitudes of FFT results. Moreover, Sum Square of Relative Errors (SSRE) of Wavelet transformation (WT) signal sequence, including approximation signals and detail signals, was introduced to evaluate the similarity and relative orientation among shapes. As a complement to FFT results, the WT results can retain more detailed information of shapes including their orientations. Besides, the geometric signatures of the second phases were extracted by image processing and then were analyzed by means of FFT and WT.

1. Introduction

Shapes with associated features such as size, shape, profile roughness (boundary irregularity) and orientation have drawn extensive attention in the scientific studies.[1–7] For example, in material studies, the shape characteristics of second phases and particles have been regarded as important factors affecting materials mechanical behavior.[8]

Features such as size, shape, profile roughness and orientation can be estimated by the equivalent diameter, shape factor, fractal dimension and Feret angle.[1,9] In the past decades, researchers have proposed various methods of studying and categorizing shape characteristics.[1,3–7,10] Hentschel and Page[1] have identified redundant descriptors for shape characteristics such as aspect ratio and the square root of form factor, with each descriptor sensitive to a different attribute of shape elongation and ruggedness, respectively. Zhang and Lu[5] have classified and reviewed a variety of methods of shape representation and description techniques from both contour-based methods and region-based methods. Through multi-scale fractal dimension, Backes and Bruno[3] have successfully represented and characterized shape contour in a dynamic evolution context. What's more, artificial neural network has been introduced by Clark et al.[11] to identify the shape features of

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plants. Still, balance between accuracy and efficiency for shape description should be met with appropriate descriptors.

Among these techniques, spectrum analysis as an effective tool with many advantages, such as being simple to compute, overcoming the problem of noise sensitivity and boundary variations, has been utilized widely in shape description.[5] Drevin[6] have used the Fourier transform results to determine roundness of particles. Kindratenko et al.[12] have applied Fourier and fractal analyses to describe the shape of micro particles from microscope images. Li et al.[13] have analyzed the irregularity of graphite nodules of ductile iron in one-, two- and three-dimensional space. Through the utilization of a Fourier–wavelet representation, Lestrel et al.[14] have characterized 2D shape features within biological sciences. Though plenty of works have been done by means of the above studies, seldom of them is related to the influence of analyzed shape orientation and shape axis ratio on spectrum analysis results.

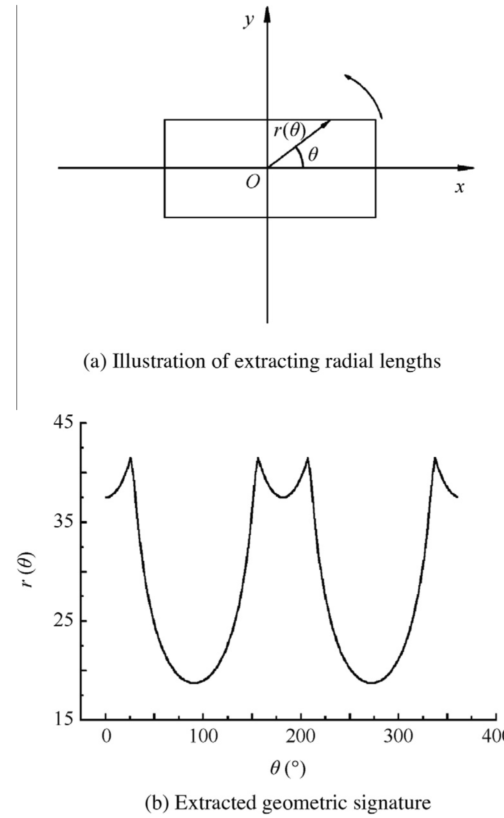
In this paper, with different types of simple shapes, shape characteristics including shape similarity, influence of orientation, shape axis ratio and profile roughness have been processed by using both Fourier transformation (FT) and Wavelet transformation (WT) in Sections 3 and 4. Besides, the application of spectrum analysis to the second phases from real microstructure morphology is shown in Section 5.

2. Methodology

2.1. Extracting geometric signature

The analysis of this study can be divided into two parts as twelve simple shapes and the second phases from real microstructure morphology. In the first part, simple shapes are selected to analyze the circle, ellipse, triangle, square and rectangle, with the same equivalent diameter ED = 50 (ED = $4\pi A/P$, P, perimeter; A, shape area). Specified shape parameters are: circle, ellipse (1:1.5) (1:1.5, meaning the ratio of length of axis or side at horizontal to vertical, the same as below), ellipse (1:2), ellipse (2:1), ellipse (1:3), equilateral triangle (triangle-1), and its rotated 60° relative to triangle-1 (triangle-2), square (square-1), and with a rotation 45° (square-2), rectangle (2:1), rectangle (1:2), and rectangle (1:3). In the second part, two second phases are from the real microstructure morphology.

As for spectrum analysis, the geometric signature should be firstly extracted from shapes.[5,6] With regard to simple shapes, their centroids are easy to be determined, while the centroids of the second phases have to be calculated by image processing (which is discussed at length in Section 5). Then, geometric signatures of shape profiles are extracted by radial vectors originating from the centroid and terminating at the boundary of the shape sweeping through 360°, starting from x axis along the counter-clockwise (as shown in Fig. 1). In this research, 128 (2^7) radial measurements are chosen with an angle increment by 2.8125°, which is prepared as shape geometric signature for the following analysis.



(a) Illustration of extracting radial lengths

(b) Extracted geometric signature

Fig. 1 Extracting geometric signature of a rectangle.

2.2. Fourier transformation (FT)

Fourier transformation is a procedure of high efficiency for spectrum analysis of a time series, especially for processing signals. In essence, it is a way to decompose the limited signal $f(t)$ to the space based on orthogonal vector $\{e^{iwt}\}$(i is the square root of -1, w is period parameter), and the transformation equations are as follows:

$$F(w_1, w_2) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} f(m,n)e^{-jw_1 m}e^{-jw_2 n} \quad (1)$$

In the above equation, w_1, w_2 and m, n are the period and space parameters, respectively. $F(w_1,w_2)$ is the Fourier space representation of $f(m,n)$ with its discrete form as

$$F(p,q) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f(m,n)e^{-j\frac{2\pi mp}{M}}e^{-j\frac{2\pi nq}{N}} \quad (2)$$

where $p = 0, 1, \ldots, M-1$; $q = 0, 1, \ldots, N-1$.

In terms of the adopted Fast Fourier Transformation (FFT) method, the efficiency of FT has been greatly improved within a wide range of application, including spectral analysis, signal processing and Fourier spectroscopy. According to FFT analysis, the amplitude can be written as follows:

$$F(u) = R(u) + jI(u) \quad (3)$$

$$A_u^S = \frac{\|\boldsymbol{F}_u^S\|}{\|\boldsymbol{F}_0^S\|} \quad u = 1, 2, \ldots, N-1 \quad (4)$$

where $R(u)$ and $I(u)$ are the real and imaginary parts of the transforming data; j is the square root of -1. A_u^S stands for the Normalization amplitude, while $\|\boldsymbol{F}_u^S\|$ indicates the amplitude of $F(u)$, and $\|\boldsymbol{F}_0^S\|$ is the amplitude of the 0th Fourier Descriptor.