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Research paper

# Stabilization of solitons in coupled nonlinear pendulums with simultaneous external and parametric excitations

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ABSTRACT

The influence of an external harmonic excitation on the intrinsic localized modes of a chain of nonlinear pendulums is numerically investigated. We show, in particular, how the existence and stability domains of solitons are modified when the coupled pendulums are simultaneously subjected to external and parametric excitations. This stabilization mechanism opens the way towards the control of the energy localization phenomena in damped nonlinear periodic lattices for efficient energy transport applications.



## 1. Introduction

In the past few years, there were several studies focused on the dynamic behavior of coupled nonlinear oscillators and the most used model is the array of coupled pendulums. This model and its properties are used to investigate many physical phenomena like the propagation of solitons on fluid surfaces [1–3], parametric generation of spin waves in ferro- and antiferromagnets [4,5], solitons in Josephson junctions [6] and microelectromechanical and intrinsic localized modes in nanoelectromechanical systems (MEMS and NEMS) [7].

The array of coupled pendulums has been studied for more than thirty years and from different points of view. Ikeda et al. [8] investigated the intrinsic localized modes for a small array of oscillators (two and three pendulums) subjected to horizontal sinusoidal excitation. The frequency response was obtained by solving the derived equation of motion using Van Der Pol's method. The authors showed the effect of the linear coupling spring an added imperfection on the soliton and they proved these results on an array of Duffing oscillators [9] subject to an external excitation. For a large number of oscillators, Ivancevic et al. [10] transformed the equations of motion of an array of coupled oscillators to one Sine-Gordon equation and reviewed the essential dynamics of a nonlinear excitation in living cellular structures. Khomeriki et al. [11] studied the tristability of a chain of pendulums driven periodically in one end and free at the other end.

There are two types of excitation: an external excitation [12,13] by applying torques to the pendulums and a parametric excitation [14–17] by moving periodically the support of the pendulums. The first type of excitation was simulated by Braiman et al. [12] on a chain of coupled damped pendulums with a free end boundary condition. The authors showed that when the chain is homogeneous (all pendulums have the same length), the oscillations become chaotic. However, their motion becomes ordered when some impurity is added. The notion of impurity was also tested numerically [13] on an array of 128 pendulums. The parametric excitation was studied numerically by Alexeeva et al. [14] and experimentally by Chen

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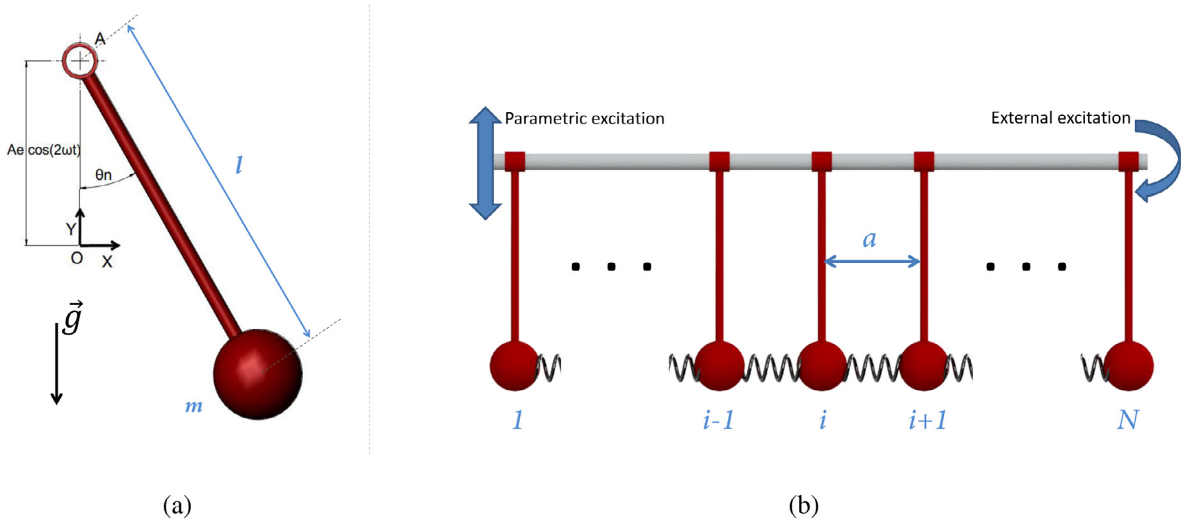


**Fig. 1.** Array of coupled pendulums under simultaneous parametric and external excitations.

et al. [15]. The authors showed that "long" impurities can extend the region of stability of the system and short impurities are responsible of oscillatory instabilities.

In term of nonlinear energy localization, the equations of motion of the system have been transformed into a nonlinear Schrodinger equation (NLS) using the multiple scale method in order to analyze numerically [16,17] and experimentally [18] the interactions between impurities and solitons in parametrically driven coupled pendulums and the impurity proved their ability to stabilize a chaotic oscillator. Nevertheless, to our knowledge, these phenomena have not been investigated when the pendulums are subjected to simultaneous excitations, even though beneficial effects have been demonstrated in term of collective dynamics for coupled nonlinear oscillators under simultaneous external and parametric excitations [19] and in terms of performances for micro and nano-sensors under simultaneous primary and superharmonic resonances [20,21].

In this paper, we derived the equations of motion describing the nonlinear dynamics of an array of coupled pendulums under simultaneous external and parametric excitations. In order to investigate the intrinsic localized modes, the system of nonlinear equations is transformed into a Schrodinger equation which has been numerically solved using the Runge-Kutta method. Numerical simulations are performed on a typical design of 301 coupled pendulums proving that the region of existence and stability of solitons can be significantly increased by means of simultaneous parametric and external excitations.

## 2. Design and model

The considered system, depicted in Fig. 1, is composed of a horizontal axle $A$. Along this axle, at equally spaced intervals, there are $N_{pen}$ equal pendulums. Each pendulum consists of a rigid rod, attached perpendicularly to the axle, with a mass $m$ at the end. At rest, all the pendulums point down the vertical. $a$ is the distance between two pendulums, $g$ is the gravity acceleration, $\theta_n$ is the angle between the $n^{th}$ pendulum and the downward vertical, $k_L$ is the linear torque constant. The kinetic and potential energy of the system can be written as:

$$V = \sum_n \frac{1}{2} k_L (\theta_n - \theta_{n+1})^2 + \frac{1}{2} k_L (\theta_n - \theta_{n-1})^2 - mgl(\sin(\theta_n) + A_e \cos(2\omega t)) \tag{1}$$

$$T = \sum_n \frac{1}{2} m v_n^2 \tag{2}$$

The potential energy $V$ includes two parts: the strain energy due to the elongation of the spring and the gravitational potential energy; $T$ is the kinetic energy due to the velocity $v_n$ of the moving mass:

$$\overrightarrow{v_n} = \overrightarrow{\dot{r}_{OA}} + \vec{\omega} \times \overrightarrow{r_{AP}} \tag{3}$$

where

$$\overrightarrow{\dot{r}_{OA}} = A_e \omega_e \sin(\omega t) \overrightarrow{y} \qquad \vec{\omega} = \dot{\theta}_n \overrightarrow{z} \tag{4}$$

$$\overrightarrow{r_{AP}} = l(\sin(\theta_n) \overrightarrow{x} + \cos(\theta_n) \overrightarrow{y}) \tag{5}$$