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Research paper

Dynamics of an advertising competition model with sales promotion

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ABSTRACT

In this paper, an advertising competition model with sales promotion is constructed and investigated. Conditions of the existence and stability of period-*T* solutions are obtained by means of the discrete map. Flip bifurcation is analyzed by using the center manifold theory and three sales promotion strategies are discussed. Example and numerical simulations are illustrated which agree well with our theoretical analysis.

1. Introduction

Advertising is a form of marketing communication used to persuade an audience to take or continue some actions, usually with respect to a commercial offering, or political or ideological support [1]. In the past few decades, many models have been developed to investigate advertising problems, such as the extended Nerlove–Arrow model [2], diffusion model [3], advertising oscillators model [4] and so on.

Advertising decisions are often made in competitive situations and have to respond to changes in market conditions across time. A differential advertising model involving the competition between two, or among three or more firms, treats competitors as players in a model and uses differential equations to model market share evolution. There have been many profound results about advertising competition in the literature [5]. For example, the Vidale–Wolfe model [6] (referred to as the V–W model thereafter) has a wide appeal because it was developed on the basis of empirical evidence and is flexible for realistic applications. By considering competitive element of a real market, Deal extended the V–W model to a duopoly [7]. The objective of the study is to maximize cumulative sales profit plus a valuation of ending market share for each player. Wang and Wu investigated a duopolistic model of dynamic competitive advertising and presented necessary and sufficient conditions for open-loop and closed-loop Nash equilibrium solutions to the model [8]. Prasad and Sethi analyzed firms' optimal advertising decisions in a duopoly market, where each firm's market share is dependent on its own and its competitor's advertising decisions. The closed-loop Nash equilibrium for the differential game between two firms was

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found [9]. Erickson extended the V–W model to an oligopoly model involving competitors' advertising efforts on attracting the untapped potential, and allowing for an expansion of the total sales of the oligopolistic competitors [10]. Analyses of symmetric and asymmetric oligopolies of the model were given to show that unit contribution and advertising effectiveness have positive effects on a competitor's own advertising and steady-state sales, while discount rate and decay rate have negative effects.

Being able to create an immediate boost in sales volume, sales promotion is an important component of a company's marketing communication strategy along with advertising, public relations, and personal selling [1,11]. Typical examples of consumer sales promotion tools include contests and sweepstakes, branded give-away merchandise, bonus-size packaging, limited-time discounts, rebates, coupons, free trials, demonstrations, and point-accumulation systems. Nair and Tarasewich addressed the optimal design of a series of promotions periodically mailed to potential customers. A methodology was presented to maximize the multiple purchases and applied to a realistic problem with experimental data [12]. Del Vecchio et al. discussed the effects of sales promotions on brand preference and found that, on average, sales promotions do not affect post-promotion brand preference by using the meta-analysis [13]. Promotion cost was considered in a model which shows that keeping the fractions of promotion cost sharing within an appropriate range increases profits for all parties [14]. Jin et al. [15] modeled a supply chain with one manufacturer and one retailer who has limited capital and faces deterministic demand depending on retail price and sales promotion, and stated that only the combination of a consignment contract with the manufacturer's right of sales promotion or a wholesale price contract with the retailer's right of sales promotion is better for both members.

Advertising and sales promotion, which are two specific marketing communications tools, account for at least 25% of UK marketing budgets [16]. However, from the above literature, very little has been known on periodic solution and its bifurcation in extended V–W models with advertising competition while sales promotion is rarely considered into a differential model. Motivated by this fact, in this study we incorporate advertising competition with sales promotion into a differential model, and restrict our attention to the complex dynamics of this model and the effect of sales promotion on sales level.

The paper is organized as follows. In the next section, advertising competition and sales promotion are considered in an extended V–W model. In Section 3, we discuss the existence and stability of periodic solutions. Bifurcation analysis of periodic solutions is presented in Section 4. Section 5 is dedicated to three sales promotion strategies. Numerical simulations on periodic solutions, bifurcation diagram, and sales promotion strategies are shown through an illustrative example in Section 6.

2. Model description

To address the dynamics of advertising decisions, Vidale and Wolfe constructed the following V–W model [6,17]:

$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} = \dot{S}(t) = ru(t)\left(1 - \frac{S(t)}{M}\right) - \rho S(t),$$

where $S(t)$ represents the sales level at time t, $u(t)$ represents the advertising expenditure at time t, r denotes the response rate to advertising, M denotes the size of the potential market or saturation level, and ρ is a decay constant.

Considering the effects of advertising competition, Deal extended the V–W model into a duopoly by using a differential system:

$$\begin{cases} \dot{S}_1 = -\rho_1 S_1 + \dfrac{r_1 u_1(t)}{M}(M - S_1 - S_2), \\ \dot{S}_2 = -\rho_2 S_2 + \dfrac{r_2 u_2(t)}{M}(M - S_1 - S_2), \end{cases} \tag{2.1}$$

where two competitors produce the same good for sale in the market, S_i denotes the sales level of competitor i $(i = 1, 2)$ at time t, ρ_i is a decay constant for competitor i, r_i represents the response rate to advertising for competitor i, $u_i(t)$ represents a variant of advertising expenditure for competitor i at time t, and M is the potential market size. System (2.1) is considered in the region $\Omega = \{(S_1, S_2) | (S_1 \geq 0, S_2 \geq 0, S_1 + S_2 \leq M\}$.

Uniform advertising policy [18], in which the firm advertises at a constant level, is applied in this study, namely, $u_i(t) = U_i$ $(i = 1, 2)$ and $t \geq 0$. Suppose that

$$B_1 > B_2 \quad \text{and} \quad \rho_1 < \rho_2, \tag{2.2}$$

where $B_i(t) = \frac{r_i U_i}{M}$ $(i = 1, 2)$. $B_1 > B_2$ means that the advertising of competitor 1 is more effective than that of competitor 2 while $\rho_1 < \rho_2$ means the decay constant rate of competitor 1 is less than that of competitor 2.

A direct calculation shows that the unique positive equilibrium $A(S_{1A}, S_{2A})$ of system (2.1) is

$$A(S_{1A}, S_{2A}) = \left(\frac{\rho_2 B_1 M}{\rho_1\rho_2 + \rho_1 B_2 + \rho_2 B_1}, \quad \frac{\rho_1 B_2 M}{\rho_1\rho_2 + \rho_1 B_2 + \rho_2 B_1}\right), \tag{2.3}$$

where $S_{1A} > S_{2A}$ under condition (2.2). The associated characteristic determinant is

$$\begin{vmatrix} \lambda + \rho_1 + B_1 & B_1 \\ B_2 & \lambda + \rho_2 + B_2 \end{vmatrix} = \lambda^2 + (\rho_1 + B_1 + \rho_2 + B_2)\lambda + \rho_1\rho_2 + \rho_1 B_2 + \rho_2 B_1,$$