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Research paper

Stochastic bifurcations in the nonlinear vibroimpact system with fractional derivative under random excitation

Yongge Yang[a], Wei Xu[a,*], Yahui Sun[b], Yanwen Xiao[a]

[a] Department of Applied Mathematics, Northwestern Polytechnical University, Xi'an, 710072, PR China
[b] State Key Laboratory for Strength and Vibration of Mechanical Structures, Xi'an Jiaotong University, Xi'an,710049, PR China

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ABSTRACT

This paper aims to investigate the stochastic bifurcations in the nonlinear vibroimpact system with fractional derivative under random excitation. Firstly, the original stochastic vibroimpact system with fractional derivative is transformed into equivalent stochastic vibroimpact system without fractional derivative. Then, the non-smooth transformation and stochastic averaging method are used to obtain the analytical solutions of the equivalent stochastic system. At last, in order to verify the effectiveness of the above mentioned approach, the van der Pol vibroimpact system with fractional derivative is worked out in detail. A very satisfactory agreement can be found between the analytical results and the numerical results. An interesting phenomenon we found in this paper is that the fractional order and fractional coefficient of the stochastic van der Pol vibroimpact system can induce the occurrence of stochastic P-bifurcation. To the best of authors' knowledge, the stochastic P-bifurcation phenomena induced by fractional order and fractional coefficient have not been found in the present available literature which studies the dynamical behaviors of stochastic system with fractional derivative under Gaussian white noise excitation.

1. Introduction

Vibroimpact system, as a specific class of nonsmooth systems, can present lots of interesting phenomena which do not exist in smooth systems, such as torus bifurcation [1], grazing bifurcation [2], sliding bifurcation [3-5]. Because of the extensive existence of random factors, it is necessary to study the vibroimpact systems excited by random factors. A lot of articles dealing with stochastic vibroimpact systems are available. Huang [6] focused on the stochastic responses of a multi-degree-of-freedom vibroimpact system excited by white noise. Zhu [7] investigated the stochastic response of nonlinear systems using the exponential-polynomial closure (EPC) method. Li [8] investigated the stochastic response of a Duffing-van der Pol vibroimpact system under correlated Gaussian white noise. Yang [9] considered the random vibrations of Rayleigh vibroimpact oscillator under Poisson white noise. Dimentberg [10] presented an excellent review on the development of vibroimpact systems subject to random perturbations.

Except for the vibroimpact system, another research hotspot which have been attracting much attention in the scientific community is the fractional calculus. It has shown [11] that the fractional-order models are more suitable than the classical integer-order models for the descripiton of various materials which has memory property. Many excellent methods have been developed by researchers to deal with fractional stochastic systems. A frequency-domain approach was presented by

* Corresponding author.
E-mail address: weixu@nwpu.edu.cn (W. Xu).

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Spanos and Zeldin [12] to investigate the random vibration of stochastic systems with fractional derivatives. The stochastic averaging method was first used by Huang and Jin [13] to investigate the stationary response and stability of a SDOF strongly nonlinear system with fractional derivative under random perturbations; Then, Chen used the stochastic averaging method to investigate many dynamical behaviors of the fractional stochastic systems, such as the stationary responses [14,15], stochastic jump and bifurcation [16], first passage failure [17] and fractional control [18,19]; Yang [20,21] explored the stochastic response of nonlinear system with Caputo-type fractional derivative subject to Gaussian white noise. An approximate analytical Wiener path integral technique put forward by Kougioumtzoglou and Spanos [22] was generalized by Di Matteo [23] to explore the stochastic response of stochastic systems with fractional derivatives terms.

From the above discussion, we can find that the dynamical behaviors presented by the stochastic vibroimpact systems or the stochastic fractional systems are complex. Thus, the dynamical behaviors of nonlinear stochastic system with both vibroimpact factors and fractional derivative must be more richer. To the best of authors' knowledge, very little work has been dedicated to the study of stochastic vibroimpact systems with fractional derivative. So it is necessary to study the stochastic response of nonlinear vibroimpact system with fractional derivative excited by Gaussian white noise.

The rest of this paper is organized as follows. The original stochastic vibroimpact system with fractional derivative is transformed into stochastic vibroimpact system without fractional derivative in Section 2. The non-smooth transformation and stochastic averaging method are used to obtain the analytical solutions of the equivalent stochastic system in Sections 3 and 4. In order to verify the effectiveness of the above mentioned approach, the van der Pol vibroimpact system with fractional derivative is worked out in detail in Section 5.1.The stochastic bifurcations are explored in Section 5.2. The conclusions are presented in Section 6.

2. Equivalent vibroimpact system

Consider the vibroimpact system with fractional derivative and excited by Gaussian white noise. The motion of the vibroimpact system is governed by the following differential equation:

$$\begin{aligned} &\ddot{x} + \varepsilon\beta_1 D^\alpha x + \varepsilon\beta_2 h(x,\dot{x})\dot{x} + \omega_0^2 x = \varepsilon^{1/2}\xi(t) && x > 0 \\ &\dot{x}_+ = -r\dot{x}_- && x = 0 \end{aligned} \tag{1}$$

where ε is a small positive parameter, $\beta_1, \beta_2, \omega_0$ are constant coefficients, $h(x,\dot{x})$ is a function of x and $\dot{x}$, $\xi(t)$ is Gaussian white noise with zero mean and correlation function $E[\xi(t)\xi(t+\tau)] = 2D\delta(\tau)$,$r$ is the restitution coefficient, $\dot{x}_+$ and $\dot{x}_-$ denote the velocities of the system after and before the impact, respectively. The value of r reflect the degree of energy loss of the vibroimpact system when collision occurs. In other words, the energy loss is extremely small when r approaches to 1, while it is extremely large when r approaches to 0.

$D^\alpha x$ denotes the fractional derivative in the Caputo sense and has the following form:

$$D^\alpha x = \frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{\dot{x}(u)}{(t-u)^\alpha}du, \ (0 < \alpha \le 1) \tag{2}$$

in which $\alpha$is the fractional order, $\Gamma(s) = \int_0^{+\infty} e^{-t}t^{s-1}dt$ is the Gamma function.

As shown in Refs [14,24], the term associated with fractional derivative serves as the classical damping force and the restoring force. According to Refs [21,24], such term can be replaced by a linear damping force and a linear restoring force:

$$D^\alpha x = \omega_0^{\alpha-1}\sin\frac{\alpha\pi}{2}\dot{x} + \omega_0^\alpha\cos\frac{\alpha\pi}{2}x \tag{3}$$

Substituting Eq. (3) into Eq. (1), the following equivalent vibroimpact system without fractional derivative can be obtained:

$$\begin{aligned} &\ddot{x} + \left[\varepsilon\beta_2 h(x,\dot{x}) + \varepsilon\beta_1\omega_0^{\alpha-1}\sin\frac{\alpha\pi}{2}\right]\dot{x} + \omega_1^2 x = \varepsilon^{1/2}\xi(t) && x > 0 \\ &\dot{x}_+ = -r\dot{x}_- && x = 0 \end{aligned} \tag{4}$$

where $\omega_1^2 = \omega_0^2 + \varepsilon\beta_1\omega_0^\alpha\cos\frac{\alpha\pi}{2}$.

3. Non-smooth transformation

According to Refs [10,25], the non-smooth transformations of the state variables are introduced as follows:

$$x = x_1 = |y|, \quad \dot{x} = x_2 = \dot{y}\mathrm{sgn}(y) \quad \ddot{x} = \ddot{y}sgn(y) \ , \tag{5}$$

where $\mathrm{sgn}(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$.

Obviously, the transformation of Eq. (5) maps the domain of the original plane$(x,\dot{x})$ onto the whole phase plane$(y,\dot{y})$. After the Eq. (5) is introduced into Eq. (4), the equations of new variables can be obtained as follows

$$\ddot{y} + \left[\varepsilon\beta_2 h(y,\dot{y}) + \varepsilon\beta_1\omega_0^{\alpha-1}\sin\frac{\alpha\pi}{2}\right]\dot{y} + \omega_1^2 y = \varepsilon^{1/2}\mathrm{sgn}(y)\xi(t) \quad t \ne t_* \tag{6a}$$