Commun Nonlinear Sci Numer Simulat 42 (2017) 93–105

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

The superposition solitons for 3-coupled nonlinear Schrödinger equations

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ARTICLE INFO

Article history:
Received 11 November 2015
Revised 15 March 2016
Accepted 6 May 2016
Available online 7 May 2016

Keywords:
Nonlinear Schrödinger equations
Superposition solitons
Interaction
Hirota bilinear method

ABSTRACT

In this paper, a Hirota bilinear method is developed for applying to the 3-coupled nonlinear Schrödinger equations. With a reasonable assumption the exact two-superposition-one-dark (TSD) and one-bright-two-superposition (BTS) soliton solutions are constructed analytically. It shows that they can transform into general mixed (dark–bright) soliton solutions in the special conditions. Moreover, the asymptotic behavior analysis shows that the collision of TSD and BTS two solitons are all elastic.

1. Introduction

In recent years, the investigation of multicomponent nonlinear systems has received much attention [1–4]. Of special concern is the multicomponent generalization of the nonlinear Schrödinger equation [3–5]. The research of N-coupled nonlinear Schrödinger (N-CNLS) equations in mathematical and physical aspects is of comparative popular interest because these equations appear in different sides of science, such as nonlinear optics [6–8], optical communication, multicomponent Bose–Einstein condensates at zero temperature [9–11], and plasma physics [12–17]. A set of N-CNLS equations which is not integrable in general, however, it becomes integrable for specific choices of parameters [15,18]. In addition to mathematics and physics, in the context of biophysics the case $N = 3$ can be used to study the launching and propagation of solitons along the three spines of an alpha-helix in protein [14].

The dynamics of multicomponent solitary waves due to their widely range of applications encompassing science and engineering is also to be paid much attention [12]. Soliton is capable of propagating over long distances without change of shape and with negligible attenuation [1,6,12]. The soliton in CNLS are often called vector solitons, which were firstly suggested by Manakov [4]. This type of vector soliton consists of a bright soliton in one component coupled to a dark soliton in another component. Such structures have recently been observed in experiments carried out in optics and Bose–Einstein condensates. Notice that mixed bright–dark solitons were occurred as well for 3-CNLS. There are bright–bright–dark (BBD) and bright–dark–dark (BDD) soliton solutions [19] excepting all bright and dark solitons. They have interesting dynamic properties, and many research achievement about this mixed solitons have been obtained [20–22]. Recently, more interesting is superposition soliton (SPS) which is dark and bright solitons can coexist in the same component for the mixed equations, and it is studied in Refs. [23–25]. Even though there are a number of works on the soliton propagation and

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http://dx.doi.org/10.1016/j.cnsns.2016.05.011
1007-5704/

collision refer to general mixed (dark–bright) solitons respectively [26–28], results are scarce for the study on the mixed (SPS and dark or SPS and bright) soliton propagation and their collision dynamics for 3-CNLS. In this paper, we aim to study this problem by using the developed Hirota bilinear method.

We consider the set of integrable mixed 3-CNLS equations (in dimensionless form) as follows

$$iq_{j,z} + q_{j,tt} + 2\left(\sum_{l=1}^{3}\sigma_l|q_l|^2\right)q_j = 0, \quad j = 1, 2, 3, \tag{1.1}$$

where q_j is the complex amplitude of the jth component, the subscripts z and t denote the partial derivatives in regard to normalized distance and retarded time, respectively. The coefficients σ_l define the sign of the nonlinearity. In mathematics, many authors have discussed the existence of ground states and bound states of system (1.1) when $\sigma_l > 0$ [29–33]. If all σ_l's are equal to 1, the corresponding system admits bright soliton solutions. Our studied case arise for mixed signs of σ_l, i.e., $\sigma_1 = \sigma_2 = 1$, $\sigma_3 = -1$ and $\sigma_1 = 1$, $\sigma_2 = \sigma_3 = -1$. In [19], Vijayajayanthi et al. have studied the bright–dark soliton solutions for this case. But very few works have appeared in the literatures to analyze the mixed (SPS and dark or SPS and bright) soliton solutions. Then the natural question arises as to whether the interesting mixed soliton solutions exhibiting regular dynamic properties and what is the relation with the general bright–dark soliton? This is the problem we will discuss in the following sections.

The goal of this paper is to construct the novel SPS to 3-CNLS (1.1). The rest of the paper is arranged as follows. In Section 2, we introduce different form of the classical procedure of the Hirota method. Also, we construct the two-superposition-one-dark (TSD) soliton solutions and analyze the interaction of TSD two solitons. In Section 3, we get one-bright-two-superposition (BTS) soliton solutions for Eq. (1.1) and discuss collision of BTS two solitons in detail. Final Section 4 is allotted for conclusion.

2. The developed Hirota bilinear method and two-superposition-one-dark soliton solutions

In this section, we briefly outline the procedure to obtain TSD soliton solutions of the mixed 3-CNLS equations using Hirota method. We denote the soliton solution of Eq. (1.1) in which the SPS and dark solitons are split up in the two components and the remaining component, respectively. To start with, let us apply the transformation

$$q_j = \frac{G_j}{F}, \quad j = 1, 2, 3, \tag{2.1}$$

to Eq. (1.1), where G_j is arbitrary complex functions of z and t while F is a real function. Then, the set of 3-CNLS equations given by Eq. (1.1) reduces to the following set of bilinear equations:

$$0 = \hat{\mathcal{A}}_1 G_j \cdot F, \quad j = 1, 2, 3, \tag{2.2}$$

$$0 = \hat{\mathcal{A}}_2 F \cdot F - 2\left(\sigma_1|G_1|^2 + \sigma_2|G_2|^2 + \sigma_3|G_3|^2\right), \tag{2.3}$$

where $\hat{\mathcal{A}}_1 = iD_z + D_t^2 - \lambda$, $\hat{\mathcal{A}}_2 = D_t^2 - \lambda$, λ is a constant to be determined. The Hirota's bilinear operators D_z and D_t are defined as

$$D_z^m D_t^n G(z,t) \cdot F\left(z', t'\right) = \left(\frac{\partial}{\partial z} - \frac{\partial}{\partial z'}\right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^n G(z,t)F\left(z', t'\right)|_{z=z', t=t'}. \tag{2.4}$$

In the following sections, we will obtain the TSD soliton solutions by constructing the expressions of $G_j(z, t)$ $(j = 1, 2, 3)$ and $F(z, t)$ in Eqs. (2.2) and (2.3) skillfully.

2.1. One-soliton solution

In this case the SPS are separated out into two of the three components and the dark soliton appears in the remaining component. This case corresponds to the choice $\sigma_1 = \sigma_2 = 1$ and $\sigma_3 = -1$ for which Eq. (1.1) becomes

$$\begin{aligned}
&i\frac{\partial q_1}{\partial z} + \frac{\partial^2 q_1}{\partial t^2} + 2\left(|q_1|^2 + |q_2|^2 - |q_3|^2\right)q_1 = 0,\\
&i\frac{\partial q_2}{\partial z} + \frac{\partial^2 q_2}{\partial t^2} + 2\left(|q_1|^2 + |q_2|^2 - |q_3|^2\right)q_2 = 0,\\
&i\frac{\partial q_3}{\partial z} + \frac{\partial^2 q_3}{\partial t^2} + 2\left(|q_1|^2 + |q_2|^2 - |q_3|^2\right)q_3 = 0.
\end{aligned} \tag{2.5}$$

In order to obtain the novel TSD soliton solutions of Eq. (2.5), unlike the form of the classical procedure of Hirota method, here we restrict the power series expressions of $G_j(z, t)$ $(j = 1, 2, 3)$ and $F(z, t)$ as

$$G_l = g_0^{(1)}\left(1 + \chi^2 g_2\right) + (-1)^{3-l}\chi g_1, \tag{2.6}$$

$$G_3 = g_0^{(2)}\left(1 + \chi^2 g_2\right), \tag{2.7}$$