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Research paper

Mutual synchronization of oscillating pulse edges in point-coupled transmission lines with regularly spaced tunnel diodes

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ABSTRACT

In this paper, we investigate the mutual synchronization of oscillating pulse edges developed in point-coupled transmission lines periodically loaded with tunnel diodes (TDs). When supplied with an appropriate voltage at the end of a TD line, a pulse edge exhibits a spatially extended limit-cycle oscillation on the line. In this study, the properties of this mutual synchronization of edge oscillation established in two coupled TD lines are discussed. We examine the mutual synchronization using phase sensitivity calculated by applying phase-reduction scheme to the transmission equation of a TD line. The phase difference between the synchronized edges and oscillation frequency is calculated depending on the coupling cell. We then validate the reduced model via time-domain calculations of edge oscillations.

1. Introduction

Attracting attention in both science and engineering, the nonlinear wave properties in transmission lines periodically loaded with tunnel diodes (TDs), referred to as TD lines for brevity, are the focus of our current study. Because a TD line adequately simulates the nerve axon on the basis of the Hodgkin–Huxley model, it is used in the field of physiology to characterize electrical pulses propagating across a nerve axon [1,2]. Use of a TD line has also been investigated in high-speed electronics. For high frequency usage, each TD is connected by a series inductor. Setting the appropriate biasing voltage and current for the line generates a steep incident pulse edge by the loaded TDs [3–5]. Several schemes employing TD lines to generate short electrical pulses have been reported to date [6,7].

In addition, it has been observed that a voltage edge repeatedly turns around halfway on a TD line with an appropriate boundary condition [8]. Here, the edge oscillation is shown to be a limit cycle that exhibits several synchronization phenomena. Recently, we investigated the external synchronization of the oscillating edge(s) on a TD line using numerical and experimental methods [9]. The predictions made by the phase-evolution equation were validated by the experimental observations, which made it natural to consider the two edge oscillations that are mutually synchronized. Because the edge oscillations develop on a spatially extended platform, synchronized properties such as phase difference and frequency shifts may depend on the cell at which the TD lines couple. We consider point-coupled TD lines, i.e., the two TD lines coupled via a single cell, as one of the most elemental systems exhibiting mutual synchronization.

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The main motivation to consider the mutually synchronized edge oscillations is their potential to yield low-noise oscillators with controlled phases. For the oscillators used in phased-array systems, their phases have to be precisely controlled. We will see that the phase difference between two edge oscillations can be controlled by the positions of the connecting cells of supporting TD lines. In addition, the edge oscillations are expected to exhibit significantly low-phase-noise property among various TD line networks including loopback topology. Furthermore, the oscillation frequency of an edge oscillation is controlled via the applied voltage. These distinguishing properties of an edge oscillation lead to the development of aforementioned high-performance oscillators. On the other hand, it is equally important to examine the contribution of oscillator's internal degrees of freedom to the establishment of synchronization. It has been shown that the weakly coupled oscillators are well characterized by the phase model, which represents each oscillator by the unique phase variable. As the coupling strength increases, the oscillation amplitude starts to contribute to the synchronized dynamics like oscillation quenching. It is worthwhile to investigate how the large degrees of freedom consisting of an edge oscillation contribute to the establishment of synchronization under the influences of strong couplings. The present study secures the baseline to discuss the synchronization properties beyond the phase model.

The phase difference between the two edge oscillations and the frequency shift owing to the mutual synchronization are then the primary properties to be characterized; we examined their dependence on the position of the connected cells. From the phase-reduction scheme described in [10,11], we obtained phase sensitivity from the transmission equations of a TD line. In the reduced model, a limit cycle is identified only through phase sensitivity, which quantifies how the limit cycle's phase responds to perturbations. For comparison, we used another more direct and cumbersome approach that numerically solves the transmission equations of TD lines in the time domain. The phase difference and frequency shift both coincided well with the corresponding values obtained via the phase-reduction scheme. Furthermore, in contrast to the time-domain calculations, the phase-reduction scheme required time-consuming calculations only for numerically estimating the phase sensitivity. Overall, the validity of the phase-reduction scheme results in the cost-effective characterization of the properties of mutual synchronization established between two TD lines with arbitrary couplings. Due to this convenience, the phase reduction scheme gives the unique method to examine exhaustively various topologies of TD line. In addition, the phase sensitivity is also proved to quantify the phase noise [13].

In addition to this introductory section, our paper is organized as follows. In Section 2, we review the fundamental properties of edge oscillation in a TD line including the device, structure, and dynamics of edge oscillation. In Section 3, we discuss the numerical evaluation in the time domain of the mutual synchronization observed in edge oscillations on two point-coupled TD lines, whose bifurcational structure is also described. Predictions of the phase-reduction scheme are described in Section 4, as are our comparison with the time-domain calculations. Finally, we provide our conclusions and offer directions for future work in Section 5.

2. Fundamental properties of edge oscillations in a TD line

Fig. 1(a) shows the two cells of a TD line with L, R, C, and I_D representing series inductance, series resistance, shunt capacitance, and current of the shunt TD of the unit cell, respectively. The current–voltage relationship of a TD that we used for the calculations is shown in Fig. 1(b). Here, there are two characteristic voltages, namely, the peak and valley voltages, which are denoted by V_p and V_v, respectively. The TDs exhibit negative differential resistance at voltages between V_p and V_v. Any type of TD, including Esaki and resonant tunneling diodes, can be generally used as a platform to develop edge oscillation, which is the focus of our current study.

Fig. 2 illustrates typical edge oscillation behavior, in which the spatial position on the TD line is shown horizontally, whereas the voltage is shown vertically. Fig. 2(a) shows the behavior of line voltage that moves to the far end. Because of losses and leakage, the edge is gradually attenuated; it nearly disappears, as shown in Fig. 2(b). At this stage, a stable traveling front develops and starts propagating back to the near end as shown in Fig. 2(c). Once the voltage edge returns to the input end, it is reflected again to propagate from the input end as shown in Fig. 2(d). The voltage edge repeats this process, thus oscillating on the line. This edge oscillation has noteworthy properties such as voltage-controlled oscillation frequency and spatial extendedness. In what follows, we refer to *from* and *to* the input end as *forward* and *backward* directions, respectively. Here, the velocity of the voltage edge does not significantly depend on the input DC voltage. Moreover, the edge propagates further, thus increasing the turnaround time for greater input DC voltages. Therefore, the frequency of the edge oscillation decreases as the input DC voltage increases.

3. Mutual synchronization in point-coupled TD lines

Fig. 3 shows the point-coupled TD lines that we investigated. For brevity, the upper and lower TD lines are called lines 1 and 2, respectively. Given this, the M_1th cell of line 1 is coupled to the M_2th cell of line 2 through R_c. The total cell number for lines 1 and 2 is commonly set to N. Initially, all line voltages are set to zero. Then, the input end is connected to a voltage source with negligible internal resistance that outputs the DC voltage of A_i, with the other end short-circuited for line i ($i = 1, 2$). The nth voltage and current on lines i ($i = 1, 2$) are denoted as $I_{i,\,n}$ and $V_{i,\,n}$, respectively. Moreover, we define $2\,N + 1$ variables $X_{i,\,n}$ ($n = 1, \ldots, 2\,N + 1$) as $X_{i,n} = I_{i,n}$ for $n = 1, \ldots, N + 1$ and $X_{i,N+1+n} = V_{i,n}$ for $n = 1, \ldots, N$. Additionally, $L_{i,\,n}$ ($i = 1, 2$, $n = 1 \ldots \mathrm{N} + 1$) and $C_{i,\,n}$ ($i = 1, 2$, $n = 1 \ldots \mathrm{N}$) represent the line inductance and capacitance, respectively, at the nth cell of line i to incorporate the variations of reactance values. Then, the transmission equations of