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Research paper

On the discretization and control of an SEIR epidemic model with a periodic impulsive vaccination

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ABSTRACT

This paper deals with the discretization and control of an SEIR epidemic model. Such a model describes the transmission of an infectious disease among a time-varying host population. The model assumes mortality from causes related to the disease. Our study proposes a discretization method including a free-design parameter to be adjusted for guaranteeing the positivity of the resulting discrete-time model. Such a method provides a discrete-time model close to the continuous-time one without the need for the sampling period to be as small as other commonly used discretization methods require. This fact makes possible the design of impulsive vaccination control strategies with less burden of measurements and related computations if one uses the proposed instead of other discretization methods. The proposed discretization method and the impulsive vaccination strategy designed on the resulting discretized model are the main novelties of the paper. The paper includes (i) the analysis of the positivity of the obtained discrete-time SEIR model, (ii) the study of stability of the disease-free equilibrium point of a normalized version of such a discrete-time model and (iii) the existence and the attractivity of a globally asymptotically stable disease-free periodic solution under a periodic impulsive vaccination. Concretely, the exposed and infectious subpopulations asymptotically converge to zero as time tends to infinity while the normalized subpopulations of susceptible and recovered by immunization individuals oscillate in the context of such a solution. Finally, a numerical example illustrates the theoretic results.

1. Introduction

The use of mathematical models to describe the propagation of epidemic diseases within a host population has been broadly carried out for several decades [1,2]. Such models can be used to predict the evolution of the spreading of the disease. These predictions lead to take measures in order to modify the disease transmission dynamics when the spreading evolution shows a tendency of the infectious disease to become endemic within the host population. Such measures can consist of the application of vaccination strategies, quarantines, campaigns on the communication channels to make the population take behaviour patterns to prevent contagious and so on [3–8]. In this way, the propagation of the infectious disease can be reduced within the host population and, eventually, the illness can be extinguished. For instance, the research in [7] proposes a control strategy to fight against the spreading of malaria. Such a strategy consists of introducing in the

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habitat genetically modified mosquitoes for outcompeting with the wild mosquitoes and interfering with their reproductive processes. Such an action could diminish the transmission of the malaria pathogen within the human population. The authors in [8] analyse the dynamics of the human immunodeficiency virus (HIV) epidemics under the application of different treatments based on drugs therapies.

Several vaccination techniques have been proposed in the literature: constant, impulsive, time-varying to cite but a few [9–12]. Also, feedback control laws can be designed if a vaccination action depending on the model variables is proposed [13]. On the other hand, the influence of the media coverage on the transmission of infectious diseases has been studied in [14]. Concretely, such an influence is added to the model as a function that slightly modifies the transmission rate of the infection among the host population. Also, the influence of the recovered by immunization individuals on the spread of childhood infectious diseases has been studied in [15]. Furthermore, an impulsive vaccination strategy with saturation effects in the vaccination rate has been proposed in [16]. Such a saturation takes into account the limited medical resources such as a limited number of vaccines in an emerging infection disease, for instance.

A great variety of models have been used to study the propagation of infectious diseases. An important kind of such models is referred as the class of compartmental models [17]. Such models split the total population in different categories depending of the status of the individuals with respect to the infection. One of the models of this type is the known SEIR epidemic model. Such model splits the population in four categories: susceptible (S), exposed (E), infectious (I) and recovered (R) by immunization subpopulations. The total population of the model can be considered as a constant if the mortality caused by the infectious disease and/or the disease time duration are sufficiently small so that such an assumption be acceptable. Otherwise, a time-varying population has to be considered to study properly the dynamics of the infectious disease [18]. The propagation of infectious diseases can be analysed by using continuous-time or discrete-time models. In the later case, the disease propagation can be modelled directly in the discrete-time domain [19] or the discrete-time model can be derived from a discretization of the continuous-time one [20–25].

The use of discrete-time models is interesting since measurements related to epidemic statistics take place, for practical reasons, at fixed intervals independently of the characteristics times of infection processes. This fact makes easier to parameterize a discrete-time than a continuous-time epidemic model. Moreover, the design of a control strategy based on the current evolution of the disease within the host population requires the measurement of the number of individuals in each subpopulation category. As a consequence, the use of discrete-time instead of continuous-time models is also preferred since the amount of necessary computation effort to synthesize the control law can be considerably reduced since such measurements are done only at the sampling instants. Both features, namely, the epidemic model parameterization and the reduction of computational effort to synthesize control actions, are the advantages of using discrete-time instead of continuous-time models to describe the spreading of infectious diseases under control strategies.

Motivated by the aforementioned features an analysis of a SEIR discrete-time epidemic model with time-varying population under the application of a periodic impulsive vaccination strategy is carried out in this paper. Such a model is obtained from a discretization of an original continuous-time SEIR model. Discretization of continuous-time epidemic models has been previously dealt in the literature [20–25]. The paper in [20] applies a nonstandard finite difference (NSFD) discretization method to a continuous-time SIR epidemic model of childhood diseases with constant vaccination strategy. Such a discretization method provides a discrete-time model with the same equilibrium points as those of the continuous model and the properties about global asymptotic stability are consistent with the continuous model for any size of numerical time-step. The authors in [21] propose a class of discrete-time SIR epidemic models which are derived for continuous-time SIR epidemic models with distributed delays by using a variation of the backward Euler method [26]. The reference [22] deals with two variants of the continuous-time SIR model treating time as a discrete variable while maintaining the features of the original model. The first variant uses a discretization method which provides a second-order discrete-time model. The second one is a kind of generalized cellular automaton obtained following the ultra-discretization procedure introduced by Tokyo-Kyoto group some years ago [23].

Our work in [24] uses the forward Euler method to obtain a discrete-time SEIR epidemic model and the analysis of the model dynamics under the application of a periodic impulsive vaccination is carried out. However, the discretization method proposed in the current paper provides a discrete-time model closer to the continuous-time one, for a given value smaller than 1 of the sampling period, than that obtained by using the discretization method of [24]. This fact allows the use of sampling periods larger than those used in [24] with the discrete-time model being close to the continuous-time one. This feature also implies a reduction of computational effort if a control strategy based on measurements of the model variables is applied. As a consequence, the discrete-time SEIR model obtained by applying the proposed discretization method differs from that of [24] so that the analysis of its equilibrium points and the design of an impulsive vaccination strategy must be studied with the existence and stability proofs of a disease-free periodic solution under such a strategy. Finally, our work in [25] applies the NSFD discretization method to a SEIR continuous-time model for obtaining a discrete-time one which maintains its properties of positivity and existence and stability of equilibrium points. Such a discrete-time model is used to design an asymptotically stable observer system which is capable of estimating the actual state variables of the model. However, such a paper does not deal with the design of any control strategy unlike the current paper proposes.

The paper is organized as follows. Section 2 describes the proposed discretization method which includes a free-design parameter to be adjusted for guaranteeing the positivity of the derived discrete-time model as the nature of epidemic models requires. Such a positivity property is analytically proven. Also, a study of the equilibrium points of such a discrete-time model is presented. Section 3 deals with a normalized version of the discrete-time SEIR model. Such a normalized model is