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Research paper

The general theory of the Quasi-reproducible experiments: How to describe the measured data of complex systems?

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ABSTRACT

In this paper, we suggest a general theory that enables to describe experiments associated with reproducible or quasi-reproducible data reflecting the dynamical and self-similar properties of a wide class of complex systems. Under complex system we understand a system when the model based on microscopic principles and suppositions about the nature of the matter is *absent*. This microscopic model is usually determined as “the best fit" model. The behavior of the complex system relatively to a control variable (time, frequency, wavelength, etc.) can be described in terms of the so-called *intermediate model* (IM). One can prove that the fitting parameters of the IM are associated with the amplitude-frequency response of the segment of the Prony series. The segment of the Prony series including the set of the decomposition coefficients and the set of the exponential functions (with $k=1,2,\ldots,K$) is limited by the final mode K. The exponential functions of this decomposition depend on time and are found by the original algorithm described in the paper. This approach serves as a logical continuation of the results obtained earlier in paper [Nigmatullin RR, W. Zhang and Striccoli D. General theory of experiment containing reproducible data: The reduction to an ideal experiment. Commun Nonlinear Sci Numer Simul, 27, (2015), pp 175–192] for *reproducible* experiments and includes the previous results as a partial case. In this paper, we consider a more complex case when the available data can create short samplings or exhibit some instability during the process of measurements. We give some justified evidences and conditions proving the validity of this theory for the description of a wide class of complex systems in terms of the reduced set of the fitting parameters belonging to the segment of the Prony series. The elimination of uncontrollable factors expressed in the form of the apparatus function is discussed.

To illustrate how to apply the theory and take advantage of its benefits, we consider the experimental data associated with typical working conditions of the injection system

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in a common rail diesel engine. In particular, the flow rate of the injected fuel is considered at different reference rail pressures. The measured data are treated by the proposed algorithm to verify the adherence to the proposed general theory. The obtained results demonstrate the undoubted effectiveness of the proposed theory.

1. Introduction and formulation of the problem

The subject associated with treatment of data of different nature and from different fields or applications is today considered as well developed. It includes many books [2–10], a massive amount of papers (that cannot be enumerated here) and numerous conferences; they create a clearly expressed trend that is supported by many researches working in this field. The directed trend can be simply formulated: if an experimentalist has a set of accurately measured data obtained from a reliable equipment, then the problem of a theoretician is to find the microscopic/empirical model and to describe these data (after elimination of the influence of the apparatus function and random fluctuations) in terms of the fitting parameters that follow from the "best-fit" (microscopic) model. This is the basic tendency that forms a specific interaction between any theory and experiment and many researches follow and support this paradigm. In particular, non-trivial fractal model for description of the averaged motions in mesoscale was found [11]. It enables to describe a wide set of blow-like signals that can arise in many complex systems. Here we should mark the recent review paper [12] totally dedicated to consideration of different methods used in analysis of complex systems. The active interest in investigation of properties of many complex systems where the microscopic model cannot be created allows us to put forward the following problem: *is it possible to formulate a general theory based on an intermediate model (IM) for description of the properties of a wide class of complex systems in one unified scheme?* This intermediate model should present a "universal" platform containing the reduced set of the fitting parameters which can describe the measured data of the considered complex system with high accuracy in order to compare different responses (measured data) in one unified scheme. If this platform can be created, then it will present an interest for the theoreticians as well. Namely, any theory (or suggested microscopic model) expressed in the parameters of the IM will be compared with experimental data that are expressed also in the *same* set of the fitting parameters. This unified scheme detects possible errors that are admitted by both sides to achieve the true coincidence between the competitive theory and data cleaned from the influence of the apparatus and other distortions.

We want to prove that an "ideal" IM can be presented by the segment of the Fourier series, F-series for short, in the case of the "ideal" experiment [1], or by the segment of the generalized Prony series. Below, it will be proved that the IM can be presented by a function of the type

$$F(t) = [\kappa_1(t)]^{t/T}\mathrm{Pr}_1(t) + [\kappa_2(t)]^{t/T}\mathrm{Pr}_2(t), \tag{1}$$

where T is a mean period of time between two successive measurements, $\mathrm{Pr}_{1,2}(t)$ and $\kappa_{1,2}(t)$ are periodic functions with respect to the value of T. These functions can be presented by the segments of the F-series

$$\begin{aligned}&\mathrm{Pr}_{1,2}(t\pm T) = \mathrm{Pr}_{1,2}(t),\ \ \kappa_{1,2}(t\pm T) = \kappa_{1,2}(t),\\ &\Phi(t) = A_0 + \sum_{k=1}^{K}\left(Ac_k\cos\left(2\pi k\frac{t}{T}\right) + As_k\sin\left(2\pi k\frac{t}{T}\right)\right).\end{aligned} \tag{2}$$

The function $\Phi(t)$ is associated with any periodic function figuring in expression (1). The coefficients of this F-series (including the coefficient A_0, the mean value of T and the value of the final mode K) form the set of the desired fitting parameters (Ac_k, As_k, $k=1,2,\ldots,K$) that can describe the response of the analyzed complex system. The periodic functions $\kappa_{1,2}(t)$, under reasonable suppositions described below, express the influence of the apparatus function and the procedure of its elimination reduces the IM to the segment of F-series

$$F(t) = \mathrm{Pr}_1(t) + \mathrm{Pr}_2(t),\ \ F(t\pm T) = F(t), \tag{3}$$

that corresponds to an "ideal" experiment, when all measurements are becoming *identical* to each other. This problem is a logic continuation of the previous investigation that produced the results shown in paper [1], where the approximate expression (1) for the case $\kappa_1(t) = \lambda_1,\ \kappa_2(t) = \lambda_2,\ \lambda_{1,2} = const$ was obtained. In comparison with Eq. (12) considered in [1] (see also Eq. (4) below), we put the constant $b=0$ and consider the simple case $L=2$, when the memory between measurements is short. The Eq. (1) describes also a more complex case, when the measured data are *quasi-reproducible* (i.e. they can be varied during the measuring process). Earlier, it was supposed that these measurements are relatively stable (reproducible). The applicability of this general theory and its possible limitations will be discussed in the last section.

The structure of this paper is organized as follows. In the second section, we give the basics of the new theory, the third section is related to the description of important details associated with experimental measurements that are used for confirmation of this theory. The fourth section contains the algorithm and describes the basic treatment stages that can be also applied to data obtained from other complex systems. The fifth section is related to discussion of the obtained results and to further steps that will be useful and constructive for further research.