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Research paper

Characterization of intermittency at the onset of turbulence in the forced and damped nonlinear Schrödinger equation

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In this paper we characterized intermittent transitions from temporal chaos to turbulence in the forced and damped nonlinear Schrödinger equation. We demonstrate using finite time Lyapunov exponents that during the transition a fraction of unstable periodic orbits embedded in a low dimensional chaotic attractor loses transversal stability, in a way that nearby trajectories are expelled away from its vicinity (a mechanism referred to as intermittency induced by Unstable Dimension Variability). During the transition, an appropriate decomposition of the Fourier phase space into transversal and longitudinal modes is performed. The analysis of modes dynamics sheds new light in the understanding of intermittency in spatially extended dynamical systems. Subsequently a perturbation is applied to the system in order to control the intermittent extreme events and reduce their occurrence.

1. Introduction

Intermittent phenomena are of extreme relevance in many different areas of science, ranging from turbulent fluctuations of fluids and plasmas [1,2] to the behavior of simple electronic circuits [3,4], or even biological systems [5,6], neurological networks and so forth. Understanding such phenomena is therefore a very important issue in non-linear dynamics theory. The possibility to predict and control some of these intermittent phenomena initiated a very active research area [3,7–9]. Among these various possible scenarios where intermittency is present, one of the most interesting is when you have bursts of intense turbulence activity midst periods of regular flow in fluids and waves [1,8,10].

The long-standing problem of turbulence poses as one of the oldest and most challenging problems of mathematical physics [11]. The concept of turbulence has spread across the field of fluid dynamics, and today it has applications throughout many different fields of physics [12]. In the last century, after works of Lorenz [13] and Ruelle and Takens [14], a close relationship between turbulence and chaos theory has been established. In a way that many advancements have been made thanks to the new paradigm of chaos theory [12].

In that perspective, turbulence may be thought as a chaotic solution that excites a large enough number of Fourier modes [15,16]. Assuming a Fourier phase space — where each axis represents the amplitude of a given Fourier mode — turbulence can be simply represented as a chaotic attractor living in a finite subset of the formally infinite phase space.

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The onset of turbulence is one of the most interesting problems of modern physics [12]. Recent advances in non-linear systems theory has shed a new light on these classical problems. So the onset of turbulence can now be seen as a bifurcation in parameter space that gives rise to a new chaotic attractor of some higher dimensionality in Fourier phase space [14,17].

The intermittency scenario happens when a parameter is changed in a wave or fluid system close to a bifurcation that leads to turbulence [18–20]. In this case, the system dynamics shifts frequently between the former stable state - - usually regular – and a turbulent state [15]. The mechanism underlying this kind of intermittency may change depending on the particularities of the system, but one of the most prominent ones is the so called *on-off intermittency induced by Unstable Dimension Variability* [10,16,21,22], which will be explored in this work.

Among the many different classes of models that allow the study of wave turbulence and intermittency, one-dimensional PDEs stand as an appropriate choice, as they have interesting mathematical characteristics and also have relative computational low-cost. Among different available PDEs with physical interest, the one-dimensional Nonlinear Schrödinger equation is a largely used model [23–26], as it has applications in many different areas of physics. Provided that an appropriate change of variables is made [25,27–30], the NLSE with a periodic forcing and linear damping is written as:

$$i\psi_t - \psi_{xx} - \left(g|\psi|^2 - \Omega^2\right)\psi = \epsilon - i\gamma\psi, \tag{1}$$

were ϵ is the forcing amplitude, Ω^2 is the forcing frequency and γ is the damping rate. The values used for simulation were $\gamma = 0.01,\ \Omega^2 = 0.45,\ g = 2.0$ and $2\pi/\ell = 0.9,$ where ℓ is the integration box. Finally $\epsilon \in [0.1, 0.45]$ [25]. The equation was solved numerically using the pseudo-spectral method [31,32] and a 12^{th} order predictor-corrector Adams integrator [33], with time step $h = 10^{-2}$, and a total of $N = 64$ Fourier modes were simulated.

When both ϵ and γ are null, this equation possesses an infinite number of conserved integral quantities, among them:

$$M = \frac{1}{\ell}\int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} |\psi|^2 dx, \tag{2a}$$

$$E = \frac{1}{\ell}\int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \left(-\left|\frac{\partial\psi}{\partial x}\right|^2 + \frac{g}{2}|\psi|^4 - \Omega^2|\psi|^2\right)dx, \tag{2b}$$

$$H = \frac{1}{\ell}\int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \left(\left|\frac{\partial\psi}{\partial x}\right|^2 - \frac{g}{2}|\psi|^4 + \Omega^2|\psi|^2 - \epsilon(\psi + \psi^*)\right)dx, \tag{2c}$$

were M may be understood, in some contexts, as the total number of particles or mass of the system. E is the wave energy and H is its Hamiltonian. These integrals are constants whenever $\epsilon = \gamma = 0,$ however in the non-conservative case ($\gamma \neq 0$ and $\epsilon \neq 0$) they are free to fluctuate [8]. The analysis of these integrated quantities in the study of the system dynamics is very suitable as they naturally take into account the large dimensionality of the system phase space. Therefore they are not subjected to errors due to low dimensional projection of a infinite dimensional phase space.

In this work we use the NLSE as a prototypical model for one-dimensional wave turbulence, focusing on the non-linear dynamics perspective. We show that the system's phase space can be appropriately decomposed in what we call longitudinal, mixed and transverse modes. That greatly advances the understanding of turbulent intermittency in spatially extended dynamical systems. Such that it is possible to predict and control turbulent bursts, which so far has been done only in low dimensional dynamical systems.

2. Intermittent onset of wave turbulence

The overall dynamics of the wave amplitude was analyzed as the forcing parameter (ϵ) was increased. Both the qualitative dynamics of the three dimensional profile of the wave, and the time evolution of the system integrals (Mass (M), Energy (E) and Hamiltonian (H)) were used to fully comprehend the bifurcations and changes of the dynamical behavior undergone by the system.

As the value of the forcing parameter ϵ is increased the NLSE experiences a sequence of bifurcations, from a stationary state (with ordered spatial profile) to temporal chaos, to spatiotemporal chaos (or turbulence). This last transition being non-localized in the parameter space is of great interest due to its intermittent nature.

For small values of ϵ the system has a steady state solution, as depicted in Fig. 1. The integrals of the system have a constant value in time, and the spatiotemporal profile is ordered, with only a few excited Fourier modes, and with a constant amplitude in time. From a Fourier phase space perspective, these solutions correspond to fixed points, as the amplitudes of the modes remain constant in time, and so do the integrals of the system.

Increasing the value of the forcing amplitude leads the system to a temporal chaos state, which is a dynamical state where the fluctuations of the amplitude of the wave are chaotic, even though the system still has an ordered spatial profile, exciting only a few Fourier modes. For $\epsilon = 0.21$ the amplitudes of the wave integrals are chaotic, as can be seen in Fig. 2(a). Also the spatial profile excites a larger number of Fourier modes than the previous case, even though there are still not enough modes involved in the dynamics to characterize the system as turbulent. These features can be seen in the spatiotemporal profile of the wave amplitude, depicted in Fig. 2(b). This kind of solution is better understood in the Fourier