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Research paper

Vibration analysis of nonlinear systems with the bilinear hysteretic oscillator by using incremental harmonic balance method

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ABSTRACT

This paper considers dynamics of bilinear hysteretic systems, which are widely used for vibration control and vibration absorption such as magneto-rheological damper, metal-rubber. The method of incremental harmonic balance (IHB) technique that hysteresis is considered in the corrective term is improved in order to determine periodic solutions of bilinear hysteretic systems. The improved continuation method called two points tracing algorithm which is stable to the turning point makes the calculation more efficient for tracing amplitude-frequency response. Precise Hsu's method for analysing the stability of periodic solutions is introduced. The effects of different parameters of bilinear hysteretic oscillator on the response are discussed numerically. Some numerical simulations of considered bilinear hysteretic systems, including a single DOF and a 2DOF system, are effectively obtained by the modified IHB method and the results compare very well with the 4-oder Runge-Kutta method.

1. Introduction

Recent theoretical [1–4] and experimental [5–7] researches of systems comprising substructures and nonlinear attachments provided validation of nonlinear vibration absorption and mitigation applied to mechanical engineering. Numerous novel designs [8–10] have been proposed to bring more efficient and robust nonlinear vibration absorbers by purposeful transformation of nonlinearities. Nonlinear systems that involve hysteresis such as magneto-rheological damper, metal-rubber and micro-sliding friction are more common and need to be investigated for the safety and reliability of systems.

The bilinear model of hysteresis is used widely in many practical applications, like bolted structures, beam-column connections and relay oscillators [11,12]. Some materials exhibiting hysteretic behaviour under severe loading are included. Hysteretic vibration absorbers are deemed promising for vibration reduction because of their inherent hysteretic and damping characteristics. Usually, several classical mathematical models, including Bouc Wen, Duhem, Preisach and Maxwell-Slip, are available for different hysteresis (such as smart materials, shape-memory alloy and piezoceramics) [12–14]. More mathematical models of hysteresis including discontinuous hysteresis, a hysteretic transport equation, a quasi-linear parabolic PDE (partial differential equations) and so on can be obtained in [15] in details. However, a challenging task of hysteretic systems is to find appropriate techniques capable of studying the actual nonlinear mechanisms. Under certain conditions, lots of approximate and numerical methods are proposed and developed to capture the structural vibration and response of hysteretic systems in recent years.

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Broadly speaking, the bilinear hysteresis belongs to a class of non-smooth dynamical systems which are encountered in many fields and are more wide. Therefore, many methods used in non-smooth systems such as a non-interior path-following algorithm proposed for linear complementarity problem [16], a smoothing Levenberg Marquardt method for non-linear complementarity problem [17], Morse theory of attractors described by differential inclusions [18], can be extended to hysteresis. For a class of the linear complementarity problem, it can be solvable with Lemke's complementary pivot algorithm without interior point, called as a non-interior path following method [19]. Nevertheless, robustness of this approach needs to be improved. Ma [17,20]etc. applied a smoothing Levenberg Marquardt method developed in smoothing Newton methods to the nonlinear complementarity problem and found that the proposed approach is the global convergence with conditions that the level set of problems is compact and the local quadratic convergence of the method under some assumptions.

For the friction-induced stick-slip vibrations which can be described by a set-valued friction models or dry friction, a common method to solve problems for systems with set-valued friction is used to direct friction compensation techniques [21]. Bruin [22] presented a generalization of the Popov-like criterion, which is robust to uncertainties, applied to set-valued nonlinear systems or structure with non-collocation of actuation and set-valued friction laws, they also have performed some experimental work for stabilization of systems. Unfortunately, the effectiveness of the proposed approach subjecting to many constraints, and the method needs to improve further. Hysteretic models described by differential inclusions also draw attention in the references [11,23].

Many mathematical models, instead of the non-smooth dynamical oscillator, are widely used to vibration absorption and mitigation. The methods proposed for those models are relatively mature and develop rapidly. In order to capture the properties in various hysteretic system, the models of hysteresis can be classified into two types: operator-based and differential-based [12].

Nonlinear characteristics of Bouc-Wen hysteretic systems subjected to external harmonic excitation and parametric excitation have been investigated by employing the method of multiple scales [24,25]. The first and second order approximate solutions are obtained to analyse resonant characteristics and nonlinear phenomena of hysteresis. It is undeniable that the solutions for the case of primary and secondary resonance are obtained effectively, while it hard to apply to the higher order solutions and strongly nonlinearity. Similarly, the method of equivalent linearization of hysteretic restoring force proposed in [26] is also limited to weakly nonlinearities. An analytical method for determining the periodic solutions of single degree of freedom system with an elastic-friction damping is proposed in [27]. Under the condition of constant damping in the system, a frequency analysis and stability of periodic solutions are discussed by exploiting the linearity of motion equations in each interval. As the DOF of systems changes, difficulty of analysis and calculation has dramatically increased and it is inconvenient for parameters excitation. Tuned vibration absorbers possessing hysteresis have been demonstrated by both experimental and theoretical research [28,29]. Near resonant region of vibration isolator in the magneto-rheological fluid is investigated by the modified averaging technique [30]. Both of them have pointed out that the nonlinear isolator with hysteresis has a good performance of vibration suppression.

In this paper, we focus on well-known operator-based models, namely bilinear hysteresis. Zhang [31] investigates the control problems of bilinear structures with TMD by minimizing the maximum inelastic response within a concerned frequency range. Results revealed that undesirable vibration can be alleviated by a hysteretic nonlinear substructure. The proposed method for the bilinear model can obtain the transient and steady state response of hysteretic system [32]. This technique is hard to extend to multiple degrees of freedom because of the piecewise variation of restoring force. We can figure out that the main problems of hysteretic systems are to deal with the discontinuous turning point of restoring force.

The hysteresis can be considered a class of piecewise function of restoring force. A common approach to analyse and to discuss piecewise nonlinearities is IHB method [33–36]. As we know, the IHBM has many advantages, including accuracy controllability by the numbers of harmonics, easy to use in a high-dimensional nonlinearity, than the traditional nonlinear method and is used widely in the fields of nonlinearity, including non-smooth systems. Pieree [37] investigated the dry damped systems with a multi-harmonic excitation by IHBM and found that it gets very accurate results compared with time domain solution methods. Leung [38] modified the IHBM by reversing the order of increment and linearization to obtain the bifurcated solution. However, IHB method mentioned cannot apply to the hysteresis directly because of differential term of restoring force.

The goal of the present paper is a modified IHBM of bilinear hysteretic systems that hysteresis is considered in the corrective term. An explicit iteration formulation which hysteretic restoring force is expended in Fourier series is derived. Then, a two points tracing algorithm modified from pseudo-arc-length continuation and arc-length increment method is used to capturing the resonance curves of bilinear hysteretic systems. The stability criterion of periodic solutions is studied based on precise Hsu' method [39] developed recently for Floquet theory. Similarly, hysteretic restoring force is considered in the transition matrix of Floquet theory. The effects of different parameters of bilinear hysteretic absorber in the response are studied numerically by the improved IHB method above.

The paper is outlined as follows. Description of the mathematical models of the system is explained, and an improved IHB method that hysteresis is considered in the corrective term is proposed. Meanwhile, the modified tracing algorithm and the precise Hsu's method is used to amplitude frequency response and stability analysis, respectively. In the third section, some numerical simulations and verifications are given in order to verify analytical results. The conclusions of the present study are summarized in the last section.