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Research paper

Solitons and quasi-periodic behaviors in an inhomogeneous optical fiber

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ABSTRACT

In this paper, a fifth-order variable-coefficient nonlinear Schrödinger equation for the attosecond pulses in an inhomogeneous optical fiber is studied. With the aid of auxiliary functions, we obtain the variable-coefficient Hirota bilinear equations and corresponding integrable constraints. Under those constraints, we obtain the Lax pair, conservation laws, one-, two- and three-soliton solutions via the Hirota method and symbolic computation. Soliton structures and interactions are discussed: (1) For the one soliton, we discuss the influence of the group velocity dispersion term $\alpha(x)$ and fifth-order dispersion term $\delta(x)$ on the velocities and structures of the solitons, where x is the normalized propagation along the fiber, and derive a constraint contributing to the stationary soliton; (2) For the two solitons, we analyze the interactions between them with different values of $\alpha(x)$ and $\delta(x)$, and derive the quasi-periodic formulae for three cases of the bound states: When $\alpha(x)$ and $\delta(x)$ are the linear functions of x, quasi-periodic attraction and repulsion lead to the redistribution of the energy of the two solitons, and ratios among the quasi-periods are derived; When $\alpha(x)$ and $\delta(x)$ are the quadratic functions of x, the ratios among them are also obtained; When $\alpha(x)$ and $\delta(x)$ are the periodic functions of x, bi-periodic phenomena are obtained; (3) For the three solitons, including the parabolic, cubic, periodic and stationary structures, interactions among them with different values of the $\alpha(x)$ and $\delta(x)$ are presented.

1. Introduction

Soliton-based study has been expanded into diversified fields, including fluids, plasmas, molecular biology, geology and fiber communication systems [1–9]. Optical solitons, used as the carriers of the information bits, have been one of the subjects since they are capable of propagating over long distances without losing their identities, which is a property due to the balance between the self-phase modulation and dispersion [10,11]. Theoretical prediction of an optical soliton occurring in a glass fiber has been reported [10], and experimental verification has been presented [12].

Nonlinear Schrödinger (NLS) equation is a model for the propagation and dynamics of the waves in an optical fiber [5,13]. However, the higher-order terms, such as the self-steepening, self-frequency shift, third-, fourth- and fifth-order dispersions which have not been presented there, should be taken into account when the intensity of optical field gets stronger and

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the pulses get shorter [14–22]. For instance, for the propagation of subpicosecond and femtosecond pulses in the fiber, the high-order terms are needed [23]. Effect of the fifth-order dispersion has been reported in the laser experiments when the pulses are closed to 20 fs in duration [24]. Furthermore, when the optical pulses are close to the attosecond duration in a high-intensity optical field, the fifth-order terms have been required to be considered [25–27]. An NLS equation with the fifth-order terms can be written as [28–30]

$$iu_x + \frac{1}{2}u_{tt} + u|u|^2 - i\delta\left[u_{ttttt} + 10|u|^2u_{ttt} + 10(u|u_t|^2)_t + 20u^*u_tu_{tt} + 30|u|^4u_t\right] = 0, \tag{1}$$

which describes the propagation of attosecond pulses in an optical fiber, where u represents a normalized complex amplitude of the optical envelope, the subscripts x and t respectively denote the partial derivatives with respect to the scaled distance and time, δ, a real constant, is the coefficient of the fifth-order terms, and $*$ means the complex conjugate. Lax pair of Eq. (1) has been obtained and higher-order soliton solutions have been constructed through the Darboux transformation [28]. The first- and second-order breather solutions for Eq. (1), including the first- and second-order rogue-wave solutions, have been derived [29]. Breather solution of Eq. (1) can be converted into a nonpulsating soliton solution, the conditions of which have been presented [30].

NLS-type equations with the variable coefficients have been used for the long-distance optical communications in that the varying dispersion and nonlinearity are of physical significance with the consideration of the inhomogeneities of media and nonuniformities of the boundaries [31–35]. Especially, in a realistic inhomogeneous optical fiber, characteristic parameters have been found to depend on the locations in the fiber [36]. A generalized NLS equation with variable coefficients for some soliton-management constraints has been studied [37]. A variable-coefficient higher-order NLS model for a realistic fiber of weakly dispersive and nonlinear dielectrics with distributed parameters has been addressed [38]. In the ocean and optical fiber, an inhomogeneous nonlinear Hirota equation with linear inhomogeneous coefficient and higher-order dispersion has been studied [39], where the deformed solitons, breathers and rogue waves have been obtained. A fourth-order NLS equation with linearly x-dependent coefficients for the inhomogeneous Heisenberg ferromagnetic spin chain has been probed, and the analytic one-and two-soliton solutions have been derived [40]. More variable-coefficient inhomogeneous models have been seen [41–50]. Hierarchy of the NLS equations has been said to sustain the soliton solutions due to a balance between the nonlinearity and dispersion [51–53]. Existence of the soliton solutions in some nonlinear equations due to the balance among the nonlinearity, dispersion and inhomogeneity have been reported [39,40,45,46,49,51]. Motivated by those, in this paper, we will investigate a variable-coefficient extension of Eq. (1), as

$$iu_x + \alpha(x)u_{tt} + \beta(x)u|u|^2 - i\left[\delta_1(x)u_{ttttt} + \delta_2(x)|u|^2u_{ttt} + \delta_3(x)(u|u_t|^2)_t + \delta_4(x)u^*u_tu_{tt} + \delta_5(x)|u|^4u_t\right] = 0, \tag{2}$$

which might help the investigation on the attosecond pulses in an inhomogeneous optical fiber, where the real functions $\alpha(x)$, $\beta(x)$ and $\delta_1(x)$ are respectively related to the group velocity dispersion, Kerr nonlinearity, fifth-order dispersion, and the real functions $\delta_2(x)$, $\delta_3(x)$, $\delta_4(x)$ and $\delta_5(x)$ are of the fifth-order nonlinearity terms.

To our knowledge, Lax pair, conservation laws, bilinear equations and solitons for Eq. (2) have not been investigated in the existing literatures. In this paper, we will derive the variable-coefficient Hirota bilinear equations, corresponding integrable constraints, Lax pair and conservation laws for Eq. (2) in Section 2. One-, two- and three-soliton solutions for Eq. (2) will be obtained and interaction between/among the two and three solitons will be discussed graphically in Section 3, where the quasi-period formulas for three cases will be obtained and some corresponding results will be discussed. Our conclusions will be given in Section 4.

2. Hirota bilinear equations, integrable constraints, Lax pair and conservation laws for Eq. (2)

2.1. Lax pair for Eq. (2) under constraints (6)

Through the dependent variable transformation [54,55]

$$u(x,t) = \frac{g(x,t)}{f(x,t)}, \tag{3}$$

Eq. (2) can be transformed into

$$\begin{aligned}
&i\frac{D_xg\cdot f}{f^2} + \alpha(x)\left(\frac{D_t^2g\cdot f}{f^2} - \frac{g}{f}\frac{D_t^2f\cdot f}{f^2}\right) + \beta(x)\frac{g^2g^*}{f^3} \\
&+\delta_1(x)\left[\frac{D_t^5g\cdot f}{f^2} - 10\frac{D_t^3g\cdot f}{f^2}\frac{D_t^2f\cdot f}{f^2} - 5\frac{D_tg\cdot f}{f^2}\frac{D_t^4f\cdot f}{f^2} + 30\frac{D_tg\cdot f}{f^2}\left(\frac{D_t^2f\cdot f}{f^2}\right)^2\right] \\
&+\delta_2(x)\frac{gg^*}{f^2}\left(\frac{D_t^3g\cdot f}{f^2} - 3\frac{D_tg\cdot f}{f^2}\frac{D_t^2f\cdot f}{f^2}\right) + \delta_3(x)\left(\frac{g}{f}\frac{D_tg\cdot f}{f^2}\frac{D_tg^*\cdot f}{f^2}\right)_t \\
&+\delta_4(x)\frac{g^*}{f}\frac{D_tg\cdot f}{f^2}\left(\frac{D_t^2g\cdot f}{f^2} - \frac{g}{f}\frac{D_t^2f\cdot f}{f^2}\right) + \delta_5(x)\frac{g^2(g^*)^2}{f^4}\frac{D_tg\cdot f}{f^2} = 0,
\end{aligned} \tag{4}$$