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Research paper

Moiré interferences in the map of orbits of the Mandelbrot Set

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A R T I C L E I N F O

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A B S T R A C T

This article shows the presence of Moiré Interference patterns in the map of periods of the Mandelbrot Set. It describes the requirements for their appearance and shows that such interferences are highly sensitive to the original conditions that define their calculation. The specific case herein studied shows that the Moiré interference patterns appearing in a picture of a section of the map of orbits are unpredictable, even if we obtain different maps from very similar original conditions. It begins with a brief description of the Mandelbrot Set and some of the characteristics of its orbits, the Moiré Patterns, as well as a concise introduction to a description of the Discrete Wavelet Transform. In order to develop the proposed specific case, a Multi-resolution Analysis method based on the Discrete Wavelet Transform has been used. It is significant that Moiré Interference Patterns always appear when the order of magnitudes reaches a certain limit where, what is considered as hypothetically continuous, behaves as a discrete pattern. The patterns as shown by the Wavelet analysis change drastically at the slightest modification in the original calculation conditions and it does not seem possible to predict their shape beforehand. This article ends with some conclusions and suggestions.

1. Introduction and motivation

The Mandelbrot Set shows the behavior of the Complex Numbers in quadratics operations carried on in a recursive way. The Mandelbrot Set is well known, as well as its striking graphic representation thanks to the computation tools that any computer offers nowadays.

In the light of the results of this work, the Moiré Interference Patterns turn up in a picture, either captured or created through computation, as a consequence of its digitalization or its pixel-based presentation on screen when such captured or presented picture contains some kind of grid structure.

To the best of our knowledge, there is no mathematic relationship between the Mandelbrot Set and Moiré Patterns. They are two different mathematic realities. In fact, no scientific research literature shows references to the presence of Moiré Pattern in Mandelbrot Set. However, it is possible, as shown in this article, to find Moiré interferences in some of the Mandelbrot Set representations. We wonder about the source of these findings: which grid structures account for them and which origin it may have.

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This article tries to show how and where it is possible to find some of the Moiré Interferences Patterns within the map of orbits of the Mandelbrot Set. In order to offer a self-contained and easy to read this work, it seems convenient to present an introduction to the orbits in the Mandelbrot Set, plus another one devoted to the origin and characteristics of Moiré Interferences Patterns. It is possible to approach a deep research about the presence of Moiré in Mandelbrot in two ways: (a) from a formal mathematical point of view; and (b) through the experimentation and simulation with computer programs. In this article, the second option has been chosen, and a study about a set of interference patterns with Wavelet Techniques has been done. Accordingly, a third subsection in this introduction is devoted to present some aspects of these transforms.

The article is structured as follows. After the introductory section, the concept of map of orbits within the Mandelbrot Set follows. A list of the terms used in this article, given at the beginning in this section, makes it possible to refer to the different computed images as well as to the several parts of them. This section is necessary since the Interference Moiré Pattern appears within these different maps of orbits. This section questions the origin of these interferences. A correlation between the precision of the floating-point type data and the timing of the Moiré Interference Pattern is observed. Is it possible to infer from this observed correlation that the cause of these interferences can be found in the truncation errors in the floating-point data type? Is Moire a consequence of the discretization in the pixel computation from the picture of the orbit map? Does Moiré appear sooner or later depending on the arithmetic precision considered? Would Moiré show up if the calculations were done by any technology allowing infinite precision?

Along the following sections, an answer to these questions is offered through a study case about these interference pictures. The fourth section deals with a study case. Furthermore, several Moiré Interference Patterns in the map of orbits of the Mandelbrot Set are shown. The fifth section describes the process to analyze the different sets of computed pictures: up to a total sum of eight sets, each comprising 1025 images of near the same area of the complex plane. This same section shows, in two different subsections, two procedures followed in the computing and analysis of the Moiré images of the Mandelbrot orbits map. Out of each one of these eight sets of pictures a new image of differences is obtained; and each one of these eight images is analyzed through 40 Wavelet Transforms. Afterwards, the followed procedure and its results are equally shown.

Finally, the sixth section suggests some conclusions and tasks that might be developed in the future.

2. Background

The purpose of this section is to offer a brief theoretical description of Mandelbrot Set and its orbits map, as well as about Moiré Interference Patterns according to the circumstances in which they turn up, and the conditions that foster its hypersensitivity qualities; and finally about Wavelet Transform. Unavoidably, these descriptions must be short: there is no intention to offer a complete description for every topic, but only the minimal necessary information to make it possible for the article to be self-contained. Further introductions on these topics are available in the scientific literature.

2.1. The Mandelbrot Set

There is already a long way from the appearance of Fractals. Fractals may be described as mathematical objects, the final product of the iteration of a geometrical process that is normally very simple. Such iteration, which may be infinitely recursive in theory, provides the final result (fractal) with an extraordinary apparent complexity. The most representative fractals are the Mandelbrot Set and the infinite Julia sets (Mandelbrot's predecessors) [1,2]. The appearance of fractal objects, as well as of their proliferation thanks to the computing and graphics representation power of modern computers, has contributed to the spread of a new type of geometry and to an increasing interest for it.

As is well known, to determine whether a point belongs to the M-Set or not, we take such point, square it and finally add it. Then, we square the resulting point and add the original point again repeating this same operation over and over again... If the resulting points or the intermediate values obtained from this infinite iteration are always bounded in the 2 radius circumference, then the original point belongs to the M-Set. If these results diverge towards infinity, which happens whenever a point or intermediate value is not bounded within the said 2 radius circumference, then the original point does not belong to the M-Set. Obviously, infinite iteration is not possible, and this process determines whether such point belongs to the M-Set or not only if, after a preset number of iterations, the subsequent values remain bounded in the 2 radius circumference.

The series of points resulting from the subsequent iterations performed after every original point are called Orbits. Mandelbrot's expression defines an orbit for each point in the complex plane. What we have here is a discrete dynamical system of the form $z_{n+1} = f(z_n) = z_n^2 + z_0$, where, for each point $z_0 \in \mathbb{C}$ the orbit is $\left\{z_0, z_1 = f^1(z_0), z_2 = f^2(z_0), z_3 = f^3(z_0), \cdots\right\}$ and where point z_0 belongs to the M-Set when its orbit remains bounded. This means that $f^n(z_0)$ does not tend to infinity when n tends to infinity.

Unlike traditional geometrical figures, such as circumferences, ellipses or parabolas, the M-Set, and in general any fractal object, does not use shortcuts to determine the points that are part of it. There is no analytic equation for defining the geometry of the M-set whereas the geometry of a circumference of radius r and center (x_0, y_0) actually has one. In fractal geometry, equations express an iteration, a process. One point belongs to the set defined by an equation, not when it